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Review Article

Group Decision-Making with Evaluations Bycomparative Linguistic Expressions Using Index

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Abstract: To solve the group decision-making problem with evaluations by comparative linguistic expressions, in this paper, we propose a new computation algorithm that uses the index of the set of linguistic terms built in available. Under this approach, we can easily aggregation the indexes of linguistic terms. The algorithm is strict in logic and simple in implementation because it always ensures the order of linguistic terms. We have applied the proposed algorithm on a specific application that is to make the group decide from the reviewers to choose the highest rated book. The calculation results showed the correctness and efficiency of the algorithm**.**
Keywords: Group decision-making, linguistic terms, comparative linguistic expressions.

INTRODUCTION

Making decision from the group of people is a problem that many researchers are interested in. In order to make the most reasonable decisions, we need to build a mathematical model that allows us to aggregation the evaluations of expert's opinion. Arrange and select the best option according to the evaluation criteria. This problem is also called a group decision-making model.

In previous studies (Kacprzyk, J. 1986) – (Rodriguez, R. M. *et al.*, 2011), there have been many results obtained using natural language methods. The authors gave some suggestions on approaches such as using a set of predefined language terms as suggestions for experts to use in the assessment. However, experts are limited in how they evaluate by finite set of linguistic values, so they do not always have to accurately evaluate their views. Sometimes to express an evaluation, people often use comparative language expressions. Then, the evaluation is not a single words class but can be a linguistic value domain. Aggregation must be performed on the interval, not separate values. In this paper, we give an algorithm to solve the above problem. The algorithm has been applied on a specific problem to confirm its correctness.

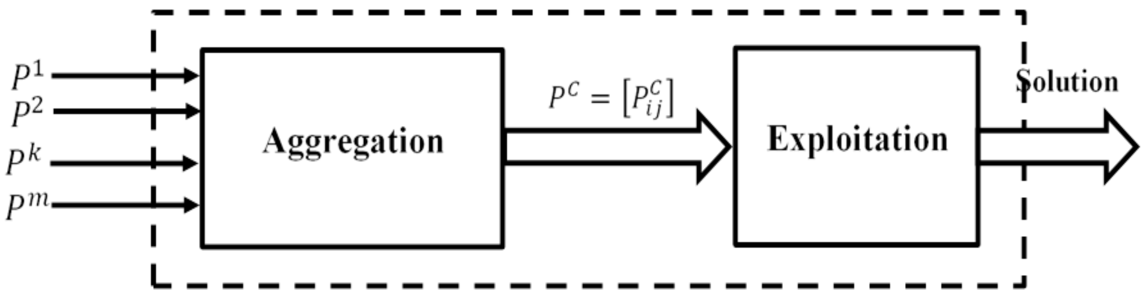

Figure.1 The general schema of the GDM

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Group Decision-Making Problem

Suppose there are m experts $E = \{e_1, e_2, ..., e_m\}$, ($m \geq 2$) evaluate n objects or solutions $X = \{x_1, x_2, ..., x_n\}$, ($n \geq 2$) information by expressing the comparison between the object x_i and x_j (Francisco, H. 2013). For example, "x_i is less than x_j", "x_i is much more than x_j", "x_i compared to x_j in the range of very few to moderate", ... Each expert e_k will give us a matrix of P^k to record the result compare with $n(n+1)/2$ objects.

$$P^k = \begin{bmatrix} p_{11}^k & p_{12}^k & \dots & p_{1n}^k \\ p_{21}^k & \dots & \dots & p_{2n}^k \\ \dots & \dots & p_{ij}^k & \dots \\ p_{n1}^k & p_{n2}^k & \dots & p_{nn}^k \end{bmatrix}; \; i,j = 1..n; k = 1..m \qquad (1)$$

Each evaluation p_{ij}^k represents the degree of opinion that evaluates the "more satisfied" relationship between the object (alternative) x_i versus x_j according to expert e_k. The problem is to aggregate these ideas in an any way that can arrange the given objects. Then, we can choose the object that is the best.

Figure 1 shows the process of collecting, processing and making decisions on the selection of objects in the group decision-making problem. In the Figure 1 scheme, it can be seen that in order to make group decisions, it is necessary to go through the following two main stages (Roubens, M. 1997):

- **Aggregation phase**:
  Collect evaluation matrices P^k from experts. Convert comparative linguistic expression to the ranges of linguistic terms for computational models. Aggregation the ranges of linguistic terms in real value based on indexes of the linguistic terms.

- **Exploitation phase**: Calculate the rating level for each object (alternative) from experts e_k. Arrange, select the object with the highest rating.

SOLVE THE GROUP DECISION-MAKING PROBLEM

Group decision-making model and computing with word

With the group decision problem, P_k is the summary matrix of expert evaluation e_k for objects. Each evaluation p_{ij}^k represents the degree of opinion that evaluates the relationship "more satisfied" (preference) between the x_i object and x_j according to the expert e_k. This "more satisfied" relationship is represented by natural language expressions. From these expressions, we need to convert the language range from which it can be easily calculated.

In order to build the linguistic information model, we must choose a set of classes from the appropriate language of their semantics. There are many approach methods to choose from. For example, a set of 7 categories from the language selected for evaluation is as follows:

$$T = \{neither, very\ low, low, medium, \\ high, very\ high, absolute\} \qquad (2)$$

The linguistic terms in this set are in order and are indexed from 0 to 6.

Our goal is to build a computational model for GDM, in which experts use the linguistic terms (or expressions) was built to evaluate objects. Then, the process of resolving the GDM problem is performed by the following steps (Herrera, F., & Herrera-Viedma, E. 2000):

Select a Set of Linguistic Terms.

- Build the comparative linguistic expressions to suggest the experts used in object evaluation.
- Collect evaluation opinions from experts.
- Convert the comparative linguistic expressions to the linguistic terms range and corresponding indexes range.
- Chose the aggregration operator on the linguistic ranges.
- Arrange and select the object with the greatest satisfaction.

Definition1. Language expression (*LE*).

For a set of linguistic terms ordered by $T = (t_1 < t_2 < \cdots < t_g)$, comparative linguistic expression structures (LE - Linguistic Express) include:

- 1) $t_i | t_i \in T$ (3)
- 2) $lower\ than\ t_i, greater\ than\ t_i$ (4)
- $at\ least\ t_i, at\ most\ t_i | t_i \in T$
- 3) $between\ t_i\ and\ t_j | t_i, t_j \in T, t_i \leq t_j$ (5)

Linguistic expressions can be in the form of $t_i \in T$ or combined by a structure of comparative linguistic expressions with other forms. Both types of expressions define a range of linguistic values and denoted by *LE*.

Thus, expert e_1 can describe the "more satisfaction" of books with comparative linguistic expressions (3) - (5) and linguistic terms in (2), such as:

$$P^1 = \begin{bmatrix} - & between\ high\ and\ very\ high & very\ high \\ at\ most\ low & - & high \\ at\ most\ low & between\ very\ low\ and\ low & - \end{bmatrix}$$

Definition2. Function of linguistic conversion *R*.

For a set of linguistic terms $T = (t_1 < t_2 < \cdots < t_g)$, $le \in LE$ are comparative linguistic expressions. Function $R: le \rightarrow R\_LE$.

- 1) $R(t_i) = [t_i, t_i], t_i \in T$ (6)
- 2) $R(at\ most\ t_i) = [t_j..t_i], j \leq i, length(t_j) = length(t_i)$ (7)
- 3) $R(lower\ than\ t_i) = [t_j..t_i], j < i, length(t_j) = length(t_i)$ (8)
- 4) $R(at\ least\ t_i) = [t_i..t_j], i \leq j, length(t_j) = length(t_i)$ (9)
- 5) $R(greater\ than\ t_i) = [t_i..t_j], i < j, length(t_j) = length(t_i)$ (10)
- 6) $T(between\ t_i\ and\ t_j) = [t_i..t_j], i < j$ (11)

Then, each linguistic interval $[t_i..t_j]$, we will have a corresponding index range $[i..j]$. the aggregation will be made on these index range.

Steps to Solve GDM Problem

Collect evaluation opinions from experts

With GDM model, there are m experts $E = \{e_1, e_2, ..., e_m\}$, $(m \geq 2)$ evaluate n objects or solutions $X = \{x_1, x_2, ..., x_n\}$, $(n \geq 2)$. We need to collect the evaluation matrix P^k as equation (1).

Convert comparative linguistic expressions to linguistic ranges

- Use the *R* function in definition 2 to convert the le∈LE expressions into language ranges$[t_i..t_j] = R(p_{ij}^k);\ p_{ij}^k \in LE; i, j = 1..n;\ t_i, t_j \in T$.
- Determine the index interval $[i..j]$ Respectively.

Aggregation

- Step 1: Use the aggregation operator Φ to combine index ranges. The result is a P^C matrix.

$$P^C = \begin{bmatrix} - & p_{12}^C & \cdots & p_{1n}^C \\ p_{21}^C & - & \cdots & p_{2n}^C \\ \cdots & \cdots & - & \cdots \\ p_{n1}^C & p_{n2}^C & \cdots & - \end{bmatrix};\ i, j = 1..n \qquad (12)$$

  Each element of $P_{ij}^C = [P_{ij}^{C-}, P_{ij}^{C+}]$ is the range of values combined from the corresponding index range.

- Step 2: Use the aggregation operator φ to combine the index ranges corresponding to the evaluation of x_i for x_l ($l = 1..n$, $l \neq i$). The result is a vector $V^R = [V_1^R, V_2^R, ... V_n^R]$. $V_i^R = [V_i^{R-}, V_i^{R+}]$ is the index range combined from m the evaluates of x_i for all other alternatives.
- Step 3: Calculate the average value of the index value range for each evaluation alternative.

Arrange, Choose Alternative

- Arrange the x_i alternatives ($i = 1...n$) in ascending (or descending) order of the prices evaluated according to the combined values.
- Select the alternative with the highest rating expressed by the combined value.

Application Problem

Suppose a GDM problem which has three experts $E = \{e_1, e_2, e_3\}$ and four books $X = \{A\_book, B\_book, C\_book, D\_book\}$. We can denote briefly that $X = \{A, B, C, D\}$.

The set of categories from the language used is equation (2). It can be abbreviated for ease of implementation as follows:

$$T = \{n(0), vl(1), l(2), m(3), h(4), vh(5), a(6)\}$$

Collect Evaluation Opinions from Experts

Evaluation matrices received from experts are as follows:

$$P^1 = \begin{bmatrix} - & at\ most\ Vl & Vh & at\ most\ Vl \\ at\ least\ Vh & - & between\ h\ and\ Vh & at\ most\ m \\ l & at\ most\ l & - & greater\ than\ h \\ at\ least\ h & greater\ than\ m & at\ most\ m & - \end{bmatrix}$$

$$P^2 = \begin{bmatrix} - & at\ most\ l & greater\ than\ m & lower\ than\ m \\ greater\ than\ m & - & h & vl \\ at\ most\ Vl & l & - & greater\ than\ h \\ between\ h\ and\ Vh & vh & between\ n\ and\ l & - \end{bmatrix}$$

$$P^3 = \begin{bmatrix} - & greater\ than\ m & between\ h\ and\ Vh & l \\ at\ most\ l & - & at\ least\ h & greate\ than\ m \\ lower\ than\ m & at\ most\ l & - & Vh \\ h & at\ most\ l & vl & - \end{bmatrix}$$

Convert Comparative Linguistic Expressions to Linguistic Ranges

Using the R function in definition 2, we get:

$$P^1 = \begin{bmatrix} - & [n, Vl] & [Vh, Vh] & [n, Vl] \\ [Vh, a] & - & [h, Vh] & [l, m] \\ [l, l] & [n, l] & - & [Vh, a] \\ [h, a] & [h, Vh] & [l, m] & - \end{bmatrix}$$

$$P^2 = \begin{bmatrix} - & [n, l] & [h, Vh] & [Vl, l] \\ [h, Vh] & - & [h, h] & [Vl, Vl] \\ [n, Vl] & [l, l] & - & [Vh, a] \\ [h, Vh] & [Vh, Vh] & [n, l] & - \end{bmatrix}$$

$$P^3 = \begin{bmatrix} - & [h, Vh] & [h, Vh] & [l, l] \\ [n, l] & - & [h, a] & [h, Vh] \\ [Vl, l] & [n, l] & - & [Vh, Vh] \\ [h, h] & [n, l] & [Vl, Vl] & - \end{bmatrix}$$

Correspondingly, we have matrices where each element is the following index ranges:

$$P^{1I} = \begin{bmatrix} - & [0,1] & [5, 5] & [0,1] \\ [5, 6] & - & [4, 5] & [2, 3] \\ [1, 1] & [0, 2] & - & [5, 6] \\ [4, 6] & [4, 5] & [2, 3] & - \end{bmatrix}$$

$$P^{2I} = \begin{bmatrix} - & [0, 1] & [4, 5] & [1, 2] \\ [4, 5] & - & [4, 4] & [1, 1] \\ [0, 1] & [2, 2] & - & [5, 6] \\ [4, 5] & [5, 5] & [0, 2] & - \end{bmatrix}$$

$$P^{3I} = \begin{bmatrix} - & [4,5] & [4,5] & [2,2] \\ [0,1] & - & [4,6] & [4,5] \\ [1,2] & [0,1] & - & [5,5] \\ [4,4] & [0,1] & [1,1] & - \end{bmatrix}$$

Aggregation

- Step 1: Use the join Φ as arithmetic mean operator. The result matrix is P^C, $P_{ij}^C = \left[P_{ij}^{C-}, P_{ij}^{C+}\right]$ is the index range that is aggregate from the 3 corresponding index ranges.

$$P_{ij}^{C-} = \frac{1}{3}\left(p_{ij}^{1-} + p_{ij}^{2-} + p_{ij}^{3-}\right)$$
$$P_{ij}^{C+} = \frac{1}{3}\left(p_{ij}^{1+} + p_{ij}^{2+} + p_{ij}^{3+}\right)$$

$$P^C = \begin{bmatrix} - & [1.33, 2.33] & [4.33, 5] & [1, 1.67] \\ [3,4] & - & [4,5] & [2.33, 3] \\ [0.67, 1.33] & [0.67, 1.67] & - & [5, 5.67] \\ [4,5\ ] & [3, 3.67] & [1,2] & - \end{bmatrix}$$

$P_{12}^C = [1.33, 2.33]$
$P_{12}^{C-} = \frac{1}{3}(0 + 0 + 4)$, $P_{12}^{C+} = \frac{1}{3}(1 + 1 + 5)$

- Step 2: Use the aggregation operator $\varphi \equiv \Phi$ to combine the index ranges corresponding to the evaluation of x_i for x_l ($l = 1..4$, $l \neq i$). The result is a vector $V^R = [V_1^R, V_2^R, \dots V_4^R]$. $V_i^R = [V_i^{R-}, V_i^{R+}]$ is the index range combined from m the evaluates of x_i for all other alternatives.

$V^R = \left[[2.22, 3], [3.11, 4], [2.11, 2.89], [2.67, 3.56]\right]$

$$V_1^{R-} = \frac{1}{3}(1.33 + 4.33 + 1)$$
$$V_1^{R+} = \frac{1}{3}(2.33 + 5 + 1.67)$$

The result V^R indicates that the index range is combined from assessments by 3 experts for each book compared to the remaining books. Table 1 shows the range of values were combined for each book.

Table. 1 The combined value ranges of the books

	A	B	C	D
V^R	V_1^R	V_2^R	V_3^R	V_4^R
	[2.22, 3]	[3.11, 4]	[2.11, 2.89]	[2.67, 3.56]

- Step 3: Calculate the average value of the combined value ranges for each book, we get $V^S = [V_1^S, V_2^S, \dots V_4^S]$.

Where: $V_i^S = \frac{1}{2}(V_i^{R-} + V_i^{R+})]$

Table.2. Average satisfaction degree of each book

	A(V_1^S)	B(V_2^S)	C(V_3^S)	D(V_4^S)
Preference	2.61	3.56	2.5	3.12

Arrange, choose alternative

From Table 2, we can arrange books that are evaluated in descending order as follows:
B > D > A > C
So, we choose the book B as the best (the highest rating).

DISCUSSION

In this paper we have proposed an algorithm to solve group decision-making problem with comparative linguistic expression. Applying the algorithm proposed solving the problem of select the best book from 4 books. As a result, a book with the highest rating is selected. The algorithm shows the correctness in terms of logic and ensures the calculation of the order of calculated values. This is a simple, intuitive approach in computing with word.

There are some issues to note:

- When collecting expert opinions, it is necessary to determine the inconsistency between the evaluations of x_i and x_j and of x_j with x_i (ie p_{ij}^k does not conflict withp_{ji}^k).
- Selecting weighted averaging aggregation operator to be able to meet the priority level in the evaluation of experts.
- It is possible to handle evaluation matrices that lack evaluation expressions.

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