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Research Article

The Kinetic and Gravitational Scaling of the Units of Electricity and Magnetism

Robert J. Buenker

Fachbereich C-Mathematik und Naturwissenschaften, Bergische Universität Wuppertal, Gaussstr. 20, D-42097, Wuppertal, Germany

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Abstract: An effective means of incorporating the time dilation effect into relativity theory is to assume that the unit of time is directly proportional to γ (u) = $(1-u^2/c^2)^{-0.5}$ on an object such as a light source that has been accelerated to speed u relative to the laboratory. In recent work it has been shown that a similar theoretical approach can be applied to other physical quantities such as length and inertial mass, and as a result, to all other mechanical properties in the mks system. This concept of *uniform scaling* can also be applied successfully for gravitational interactions. The question as to how the units of electromagnetic quantities such as electric charge and magnetic induction change with both acceleration and varying position in a gravitational field is therefore of considerable interest. Since the unit of electric charge can be chosen independently of the value of the permittivity of free space ε_0, it is shown that all electromagnetic quantities can also be assigned units *directly* in the mks system, thereby making it a trivial matter to deduce their kinetic and gravitational scaling behavior. For example, the unit of electric charge can be 1 J as long as ε_0 has units of 1 N. A table is given that makes a comprehensive comparison of the standard units in the Giorgi system with those in two such *direct mks* schemes. A simple procedure is also described for changing the numerical values of the units in a systematic manner by dividing the various electromagnetic quantities into five distinct classes. This allows one to equate the value of ε_0 to $1/4\pi$, for example, similarly as for the Gaussian system of units, while still retaining the same formulas as in the Giorgi system.

Keywords: Lorentz Force, clock-rate proportionality, Lorentz transformation (LT), alternative Global Positioning System-Lorentz transformation (GPS-LT), uniform scaling of physical properties, amended version of the Relativity Principle (RP).

I. INTRODUCTION

In recent work (Buenker, R. J. 2018) it has been shown that one can conveniently describe relativistic effects such as time and mass dilation by assuming that the units of physical quantities vary in a well defined manner depending on the state of motion of the observer and the object of measurement. A similar conclusion has been reached for gravitational interactions (Buenker, R. J. 2008a). The basic idea is simply to look upon the slowing down of clocks upon acceleration as a change in the unit of time in their current rest system (Sard, R. D. 1970a). In effect, there is a *uniform scaling* relationship between the values of a given physical quantity for two observers who are in relative motion to another and/or are at different positions in a gravitational field.

In kinematics there is an important simplifying feature in such a discussion because of the fact that all physical quantities can be expressed in terms of the units of length (m), time (s) and mass (kg). It is therefore sufficient to know how the latter three quantities scale in order to deduce the corresponding relationships for other properties such as angular momentum, energy and force. For example, since we know that the units of energy and time both increase upon acceleration to speed u in direct proportion to γ (u) = $(1-u^2/c^2)^{-0.5}$, it follows that the unit of angular momentum must scale as the square of this factor since this property is defined as the product of the latter two quantities. Similarly, the unit of frequency must scale as γ^{-1} because this quantity is defined as the reciprocal of a time period.

The fact that the speed of light is independent of the state of motion of the observer fixes the corresponding scale factor for length to also be γ, from which one can safely conclude that the *relative velocity of two objects* is the same for observers in different rest systems. The latter conclusion only holds for observers located at the same gravitational potential, however, because the rules for the uniform scaling of units also depend on one's position in a gravitational field (Buenker, R. J. 2008a). The key point is that knowledge of how the meter (m), second (s) and kilogram (kg) vary from one of state of motion to another allows one to apply this information to the scaling of all other kinematic quantities.

The situation is not so simple when it comes to the scaling of electromagnetic quantities such as charge and electric and magnetic fields, however. The fundamental equations in this field of physics are expressed in terms of a rather large set of quantities that are not directly connected to the above three units of kinematics. The Giorgi system was introduced (Reitz, J. R., & Milford, F. J. 1960a) in 1901 in order to ensure that the results of electromagnetic calculations ultimately can be expressed in terms of the above (mks) system of units, however. The question that arises in the present context is how do electromagnetic quantities such as charge, magnetic induction, electric displacement and inductance, to name just a few, vary with the state of motion of the observer or object of the measurement as well as with their respective locations in a gravitational field. In order to answer this in a definitive manner, it proves helpful to carry out an extensive review of the Giorgi system of electromagnetic units, and particularly to carefully understand how it is related through experiment and theory to the mks system.

II. Coulomb's Law and the Definition of Electric Charge

The simplest way to begin this analysis is to consider how Coulomb's Law is formulated in the Giorgi system. The force $\mathbf{F_e}$ in Newton (1 N = 1 kg m/s^2) between two electric charges q_i and q_j (expressed in Coul) separated by a distance of $\mathbf{r_{ij}}$ m is given by the vector relation:

$$\mathbf{F_e} = q_i\, q_j\, \mathbf{r_{ij}} / 4\pi\varepsilon_0\, r_{ij}^3, \qquad (1)$$

Where ε_0 is the permittivity of free space. Note that the units for ε_0 are given in such a way (Coul^2/Nm^2) so as to insure that the final result is expressed in the mks unit of force (N). The point that needs to be emphasized with regard to this equation is that it serves as a *definition of both electric charge and ε_0*. In order to satisfy the above requirement in the mks system, it is actually only necessary that the unit for the product of two electric charges q_i q_j divided by ε_0 is Nm^2. This shows that there is an *inherent redundancy* in any system of electromagnetic units *that cannot be removed by experiment.*

We can take advantage of this situation in the context of a uniform scaling procedure by defining the unit of electric charge to be some combination of mks units, that is, without introducing a new unit such as the Coulomb for this purpose. We just have to make a corresponding choice of unit for ε_0 to ensure that the force $\mathbf{F_e}$ in eq. (1) is expressed in N. For example, the unit of electric charge could be defined to be the same as for energy (1 J = 1 N m). That would simply mean that the unit of permittivity is N, since then q_i q_j / ε_0 in eq. (1) has the required unit of Nm^2 mentioned above. Since we know how N and m scale with the state of motion and position in a gravitational field (Buenker, R. J. 2018; & Buenker, R. J. 2008a), we can immediately determine how electric charge and ε_0 scale on this basis as well.

The very arbitrariness of the above choice of units might tend to make one feel skeptical about such a procedure. What it actually shows, however, is that such quantities are only defined *indirectly* by experiment. As much as we have gotten used to the idea of electrical charge over time, it should not be forgotten that there is no other way to determine its magnitude experimentally than to *measure the force exerted between it and another charge when they are a certain distance apart.*

It is no less permissible to choose a system of electromagnetic units such that ε_0 is dimensionless. This is in fact what is done with the older Gaussian set of units in which charge is expressed in esu. In that system the quantity $4\pi\varepsilon_0$ in Coulomb's Law is missing entirely. One can do this and still remain in the mks system by defining the unit of electric charge to be N$^{0.5}$m. Again, there is no *a priori* reason for avoiding such a choice because charge is only defined experimentally through eq. (1).

There is only one other relationship that must be satisfied in order to extend such an mks-type system to the description of magnetic interactions. The constant μ_0 in the law of Biot and Savart (1960b) must satisfy the equation from Maxwell's electromagnetic theory:

$$\varepsilon_0\, \mu_0\, c^2 = 1, \qquad (2)$$

Where c is the speed of light in free space (299792458 m/s). The unit in the Giorgi system is N/Amp^2 or Ns^2/Coul^2. If the unit of ε_0 is N, it follows from eq. (2) that the corresponding unit for μ_0 is s^2/Nm^2. Alternatively, if ε_0 is to be dimensionless, then the unit for μ_0 becomes s^2/m^2. That means in turn that in the latter system of units, the values of all three quantities in eq. (2) would be the same for all observers independent of their relative speed to one another (provided that they were all located at the same gravitational potential (Buenker, R. J. 2008a).

Table 1. Correlation of the units of electromagnetic quantities in various systems. The standard Giorgi system is compared with two alternatives, the Nms and $N^{0.5}$ms systems, whose units are exclusively multiples of N, m and s in the standard mks system for strictly mechanical variables. The quantities are also subdivided into K-type scaling classes, as discussed in Sect. III.

Quantity	**Symbol**	**Giorgi**	**Nms**	**$N^{0.5}$ms**	***Scaling Class***
Electric charge	q	Coul	Nm	$N^{0.5}$m	K
Permittivity	ε or ε_0	$Coul^2/Nm^2$	N	_____	K^2
Current/mmf	I	Amp	Nm/s	$N^{0.5}$m/s	K
Permeability	μ or μ_0	N/Amp^2	s^2/Nm^2	s^2/m^2	K^{-2}
Potential/emf	V	Volt	______	$N^{0.5}$	K^{-1}
Resistance/impedance	R/Z	Ohm	s/Nm	s/m	K^{-2}
Electric field	**E**	Volt/m	1/m	$N^{0.5}$/m	K^{-1}
Volume charge density	ρ	$Coul/m^3$	N/m^2	$N^{0.5}/m^2$	K
Surface charge density	σ	$Coul/m^2$	N/m	$N^{0.5}$/m	K
Electric dipole moment	$\boldsymbol{\mu}_\mathbf{e}$	mCoul	Nm^2	$N^{0.5}m^2$	K
Electric quadrupole moment	Q_{ij}	m^2 Coul	Nm^3	$N^{0.5}m^3$	K
Electric polarization	**P**	$Coul/m^2$	N/m	$N^{0.5}$/m	K
Electric displacement	**D**	$Coul/m^2$	N/m	$N^{0.5}$/m	K
Electric susceptibility	χ	Coul/mVolt	N	_____	K^2
Polarizability	α	m^2Coul/Volt	Nm^3	m^3	K^2
Coefficient of potential	p_{ij}	Volt/Coul	1/Nm	1/m	K^{-2}
Capacitance/coeff. of capacitance	C orc_{ij}	Coul/Volt	Nm	m	K^2
Current density	**J**	$Coul/m^2s$	N/ms	$N^{0.5}$/ms	K
Conductivity	g	Coul/msVolt	N/s	1/s	K^2
Resistivity	η	msVolt/Coul	s/N	s	K^{-2}
Magnetic flux	Φ	Weber	s	$N^{0.5}$s	K^{-1}
Magnetic induction	**B**	$Weber/m^2$	s/m^2	$N^{0.5}s/m^2$	K^{-1}
Magnetic vector potential	**A**	Weber/m	s/m	$N^{0.5}$s/m	K^{-1}
Magnetic scalar potential	U*	Amp	N/ms	$N^{0.5}$m/s	K
Magnetic dipole moment	**M**	m^2Amp	Nm^3s	$N^{0.5}m^3/s$	K
Magnetization	**M**	Amp/m	N/s	$N^{0.5}$/s	K
Inductance	L	Henry	s^2/Nm	s^2/m	K^{-2}
Magnetic current per unit area	$\mathbf{J_m}$	Amp/m^2	N/ms	$N^{0.5}$/ms	K
Magnetic intensity	**H**	Amp/m	N/s	$N^{0.5}$/s	K
Reluctance	*R*	Amp/Weber	Nm/s^2	m/s^2	K^2
Admittance	Y	Mho	Nm/s	m/s	K^2

Table 2. Conversion of various electromagnetic units from the Giorgi to the KNms system discussed in Sect. III (c is the speed of light in free space, 299792458 m/s).

Quantity	**Giorgi**	**KNms**
Electric charge	1 Coul	$10^{-3.5}$c Nm
Electric current	1 Amp	$10^{-3.5}$c Nm/s
$4\pi\varepsilon_0$	$10^7/c^2$ $Coul^2/Nm^2$	1 N
$\mu_0/4\pi$	10^{-7} N/Amp^2	$1/c^2$ s^2/Nm^2
Electric field	1 Volt/m	$10^{3.5}$/c 1/m
Potential	1 Volt	$10^{3.5}$/c
Magnetic induction	1 $Weber/m^2$	$10^{3.5}$/c s/m^2
Magnetic intensity	1 Amp/m	$10^{-3.5}$c N/s
Magnetic flux	1 Weber	$10^{3.5}$/c s
Electric displacement/polarization	1 $Coul/m^2$	$10^{-3.5}$c N/m
Capacitance	1 Farad=Coul/Volt	$10^{-7}c^2$ Nm
Inductance	1 Henry	$10^7/c^2$ s^2/Nm

Once the unit of electric charge has been fixed in the mks system, the corresponding units for all other quantities that occur in the theory of electricity and magnetism are determined by the standard equations in which they occur. A fairly extensive list of such quantities illustrating this point is given in Table 1. The corresponding units are always given in terms of those of force, length and time in the mks system. Two sets are given in each case, one in which the unit of charge is Nm and the other in which it is $N^{0.5}$m. The former is referred to as the Nms system so as to distinguish it from the standard mks system for purely kinematic quantities, the other as the $N^{0.5}$ms system, in which ε_0 is dimensionless.

Just a few examples will be given below which emphasize the practicality of the concepts introduced above. The unit of potential/emf U is dimensionless in the Nms system since it is proportional to electric charge and inversely proportional to ε_0 and a distance given in m. It has the unit of $N^{0.5}$ in the other system based on the same definition. Since the electric field **E** is the gradient of a potential, it follows that it has a unit of m^{-1} in the Nms system and $N^{0.5}$/m in the other. The unit of current I is Nm/s in the former case, while that of resistance R (I=V/R) is accordingly s/Nm. In the $N^{0.5}$ms system, R has the unit of s/m, i.e. the reciprocal of that of velocity, whereas the unit for I is $N^{0.5}$m/s.

In the Giorgi system of units, the magnetic force $\mathbf{F_m}$ for a given charge q moving with velocity **v** in magnetic field **B** is defined as:

$$\mathbf{F_m} = q\,\mathbf{v} \times \mathbf{B}. \qquad (3)$$

It therefore follows that **B** has the unit of s/m^2 in the Nms system and $N^{0.5}$s/m^2 in the $N^{0.5}$ms system. The Nms unit of magnetic flux (Weber in the Giorgi system) is s, consistent with the requirement that an induced emf, which is dimensionless in the Nms system of units, is given by the derivative of the magnetic flux with respect to time. In the $N^{0.5}$ms system its unit is $N^{0.5}$s. It is easy to show that the units are consistent for Maxwell's equations in both of these systems of units. For example, the differential form of Faraday's law of electromagnetic induction,

$$\mathbf{curl\ E} = -\,\partial \mathbf{B}/\partial t, \qquad (4)$$

has the units of m^{-2} on both sides in the Nms system and $N^{0.5}/m^2$ in the other.

III. A Simple Scaling Procedure for Electromagnetic Quantities

The interdependency of the definitions of electric charge q and permittivity ε_0 also presents other options for the choice of units for electromagnetic quantities than those of the Giorgi system. The esu system of units (Reitz, J. R., & Milford, F. J. 1960c) employs a much smaller unit of electric charge than Coul, for example, which therefore makes it unnecessary to include the $4\pi\varepsilon_0$ factor in eq. (1), which is to say that in this system of units, $\varepsilon_0 = 1/4\pi$. The system of atomic units, in which the electronic charge e serves as the unit of electric charge, makes the same choice for ε_0. In the present section we will illustrate how the various electromagnetic units of the Giorgi system can be modified in a systematic manner so that the latter condition is also fulfilled for mks units.

To begin this discussion it is important to note that the value of ε_0 in the Giorgi system is based directly on the speed of light in mks units: the value of $4\pi\varepsilon_0$ is equal to $10^7/c^2$. Since the speed of light in free space is no longer measured but is simply defined by international convention to have the above value, it follows that there is also no need to determine quantities such as the Coulomb (Coul) and ε_0 that are ultimately based on the value of c. A convenient quantity with which to scale the various standard Giorgi units is

$K = (4\pi\varepsilon_0)^{-0.5} = 10^{-3.5}c = 94802$. In the following we will refer to the new set of units as the KNms system. First, we define the corresponding value of the permittivity as $\varepsilon_0' = K^2\varepsilon_0$, so that $4\pi\ \varepsilon_0' = 1$ N. In general, the units in the new system are those given in Table 1 under the Nms heading, that is, with the unit of electric charge equal to 1 J = 1 Nm. It should be clear, however, that the numerical value attached to ε_0' in the new system is completely independent of this choice. One could just as well choose the unit of charge to be $N^{0.5}$m, for example, or any other combination of N, m and s, as long as one adheres to the requirements already discussed in Sect. II.

The objective in changing the numerical values of electromagnetic constants such as ε_0 is clearly to simplify computations in this important area of physics. One of the problems with changing over from the Giorgi to the Gaussian system of units is that in many cases this requires using different formulas for the same interaction. One can avoid this difficulty by agreeing at the outset that all formulas in the new KNms system will be the same as for the Giorgi system, since the latter have become standard over the past century. Let us consider eq. (1) as the first example. In order to retain the same form for this equation while using the above value for ε_0', it is simply necessary to change the numerical value of each electric charge. Specifically, one has to change the unit of charge to K^{-1} Coul. This means that the value of the electronic charge (e') becomes K times larger than the standard value in Coul, i.e, e' = 94802 x 1.602 x 10^{-19} J = 1.5187 x 10^{-14} J. In effect then, the change from the Giorgi to the KNms system of units occurs by multiplying both the numerator and denominator in eq. (1) by the same factor (K^2). The result is that one has the same form for eq. (1) as in the Gaussian or atomic unit versions, i.e, where $4\pi\varepsilon_0 = 1$ and thus does not appear explicitly.

The main point that the above discussion reveals is that it is useful to divide the variables that commonly occur in the theory of electricity and magnetism into classes according to the way in which their numerical values need to be scaled. In the KNms system, this means that each such variable simply needs to be associated with a specific power of K. This information has also been given in Table 1 in each case. Since $\varepsilon_0' = K^2\varepsilon_0$, for example, it is necessary to multiply the Giorgi value for μ_0 by K^{-2} in order to be consistent with eq. (2), that is, without changing the value of c. As a result, $\mu_0' = 4\pi/c^2$. Again, the preferred approach is not to eliminate ε_0' and μ_0' from the formulas in the KNms system, rather only to change their numerical values relative to those in the Giorgi mks system so that the form of the standard equations in the latter system is completely retained.

Other quantities that belong to the same K-class in Table 1 as electric charge are charge densities ρ and σ, dipole moment **μ**, quadrupole moment Q, current I, current density **J**, magnetic dipole moment **m**, magnetization **M** and magnetic intensity **H**. The corresponding quantities of K^{-1} type are: electric potential U or emf, electric field **E**, magnetic field (or induction) **B**, magnetic flux Φ and magnetic vector potential **A**. A check of all formulas in which the latter quantities appear shows that they always occur with counterparts in the K class mentioned first, as, for example, q and **B** in eq. (3) or q and **E** in the corresponding expression for electric force, thereby eliminating K in the overall formulas.

Some quantities do no not have to be scaled at all (K^0-type). They include all dimensionless quantities such as magnetic susceptibilities and refractive indices. The same is of course true for all non-electromagnetic quantities such as force, energy and angular momentum. A less trivial example is the Poynting vector (**E × H**), which is a product of a

K^{-1}- and K-type variable, respectively. All other commonly occurring quantities are either of K^2- or K^{-2}-type. In addition to ε_0 among the former are the dielectric constant ε and electrical susceptibility χ (Table 1), as well as polarizability, capacitance, reluctance, conductivity and admittance. Some examples of K^{-2}-type are in addition to μ_0: permeability μ, resistance, coefficient of potential p_{ij}, resistivity η and inductance L. The latter quantity is defined as dΦ/dI, which is a ratio of a K^{-1}–type quantity to the current, which is of K-type.

The conversion factors between the Giorgi and the present KNms systems of electromagnetic units for a number of the most commonly used quantities are given in Table 2. Unlike the case for the corresponding conversion between the Gaussian and Giorgi systems (Reitz, J. R., & Milford, F. J. 1960c), the formulas in which they are to be used respectively are exactly the same, as discussed above. To be specific, we have given these factors as functions of c rather than of K itself. Clearly, any other value of K could be used while still allowing the Giorgi formulas to be retained in the new system of units. The value of the electric charge in any such system of units is K times that of the numerical value in the Giorgi system ($e=1.602\times10^{-19}$). As long as one adheres to the scheme of dividing the variables into K-type classes according to the prescriptions of Table 1, this information is sufficient to characterize any new system of this type. In other words, the scaling procedure is always perfectly defined by the value chosen for K in a specific instance.

IV. Rules for Kinetic and Gravitational Scaling

In previous work (Buenker, R. J. 2018; &Buenker, R. J. 2008a) is has been shown that the properties of all objects are subject to a uniform scaling when they are either accelerated or change their position in a gravitational field. The exact nature of the scaling for a given property must be deduced from experiment in each case. For example, one knows from studies of the transverse Doppler effect that clocks slow down when they are accelerated (Sard, R. D. 1970a). Specifically, the periods of clocks and the lifetimes of metastable particles (Ayres, D. S. *et al.*, 1967) increase by a factor of $\gamma = (1-u^2/c^2)^{-0.5}$ when they are accelerated to speed u relative to their original location. A convenient way of describing this phenomenon is to assume that the unit of time is γ s in the rest frame of the accelerated object. As discussed in the above work (Buenker, R. J. 2018; &Buenker, R. J. 2008a), the units of inertial mass and length, i.e. kg and m, change in direct proportion to that of time, so this makes the mechanics of kinetic scaling particularly easy to apply.

The fundamental assumption behind such a procedure is that measurement is strictly objective, that is, *rational.* This means, for example, that when the observer has been accelerated from the same location, his measurements are affected thereby in a completely straightforward manner. If his unit of time is γ (O) s and that of the object is γ (M) s, it follows that all his timing measurements will differ from the corresponding *in situ* values by a factor of Q= γ (M) / γ (O). Moreover, the same factor applies to his measurements of the inertial mass and length of the object. We will refer to this principle of objectivity (or rationality) of measurement below as the PRM (Buenker, R. J. 2017).

The same principle allows us to scale other mechanical quantities such as angular momentum, energy and force (Buenker, R. J. 2018). One only has to know the composition of the unit of a given property in terms of m, kg and s. For example, the unit of energy is the Joule (J), which is defined in the mks system as 1 kg m^2/s^2. Since we know that each of the latter three quantities varies in direct proportion to Q as defined above, it follows that the same must hold true for this derived quantity. In turn, the unit of angular momentum, which is defined as 1 J s = 1 kg m^2/s, must scale as Q^2. In this way, all the fundamental equations of mechanics retain the same form for all observers in relative motion, even though the individual quantities contained in them differ from one observer to another.

Accordingly, neither the state of motion of the observer nor that of the object of measurement affects the measurement of certain properties. The two most prominent examples are force and velocity. Force has units of kg m/s^2, while speed or velocity is expressed in m/s. In both cases, the number of fundamental quantities in the mks system is the same in both the numerator and the denominator, so the various Q factors in the uniform scaling procedure are simply cancelled on a completely general basis. The latter fact is obviously consistent with Einstein's second postulate of the special theory of relativity (Einstein, A. (1905) that requires the speed of light in free space to be the same for all observers in relative motion, as already mentioned in the Introduction, but it goes beyond this. It states that the relative speed of any two objects is the same for all such observers (Buenker, R. J. 2015a).

Each of the above statements only holds on a general basis when the observer and the object are at the same position in a gravitational field. When this is not the case, a different set of scaling factors needs to be considered (Buenker, R. J. 2008a). The basic idea behind this aspect of uniform scaling is due to Einstein (Einstein, A. 1907). He was the first to point out that clocks run faster at higher altitude. If the difference in height is h and the acceleration due to gravity is g, then the clock at the higher gravitational potential runs A =1 + gh/c^2 times faster. Just as before with kinetic scaling, we assume that the PRM is valid in this situation as well (Buenker, R. J. 2017). If the corresponding factor is A (O) for the observer at his location in the gravitational field, while it is A (P) for the object, the ratio A (P)/A(O) must be used to multiply the *in situ* value for any elapsed time measured for the object in order to quantitatively predict the observer's findings. In previous work (Buenker, R. J. 2008a), the reciprocal of this ratio has been defined as S, and is to be used in an analogous manner as the kinetic scaling factor Q discussed above. In this case, however, distance scales as S^0 while both time and inertial mass scale as S^{-1}. Accordingly, the unit of energy scales as both Q and S (Buenker, R. J. 2008a) because of its definition as kg m^2/s^2, while the unit of force (N) scales as Q^0 and S because it has only a single factor of inertial mass in its numerator.

The main reason for converting to a system of electromagnetic units in which quantities are expressed exclusively in terms of m, kg and s is that one can use this information directly to obtain a consistent method of uniform scaling in this area of physics as well. As discussed in Sect. II, this can be done quite easily, but it must be recognized that there are many such systems possible. Two of them are described in Table 1. If one takes the Nms system, for example, the electric charge has the same unit as energy, namely 1 J=1 Nm. In this case, charge scales as Q and S in the above notation. If one employs the $N^{0.5}$ms system on the other hand, electric charge still scales as Q because N scales as Q^0 and m as Q, but as $S^{0.5}$ since N scales as S and m as S^0. The key point is that there is a redundancy in making these choices because all that is actually required is that the ratio of q^2/ε_0 has units of Nm^2 in the mks system. The latter quantity scales as Q^2 and S according to the above discussion. In the Nms system ε_0 has N as unit, whereas it is dimensionless in the $N^{0.5}$ms system, so it is seen that this requirement is met in both cases.

In any one system of electromagnetic units it thus becomes a simple matter to discern the kinetic and gravitational scaling properties of all quantities in this branch of physics. One just has to know the mks composition of a given quantity and then make use of the information of how each of these three fundamental quantities scales with Q and S. It is assumed thereby that measurements in this field, as well as all other areas of physics, are perfectly objective, that is, they must conform to the PRM discussed above (Buenker, R. J. 2017).

As intuitively reasonable as the latter principle appears to be, the fact is that it is common to see it violated in conventional discussions of relativity. This starts with the belief *that two clocks in relative motion can both be running slower than the other*. This is in clear violation of the PRM since, if true, it would mean that two observers could disagree on which elapsed time is greater for two events. The latter position has been contradicted by the experiments of Hafele and Keating (Hafele, J. C., & Keating, R. E. 1972) with circumnavigating airplanes, however, and in more recent history, by the methodology of the Global Positioning System (GPS). This work shows that clocks are slowed when they are accelerated, and that at any one time it is perfectly clear how much slower one clock is than another, regardless of who is making the comparisons (Buenker, R. J. 2008b).

Therefore, it is not only reasonable to assign definite units for all physical quantities in each rest system, it is essential to do this if one is to be able to predict with certainty the values that two observers in relative motion or at different locations in a gravitational field will measure for the same object .

V. CONCLUSION

The fact that different sets of kinetic and gravitational scaling factors for quantities such as electric charge and magnetic fields will lead to the same predictions emphasizes that there is a qualitative difference between mechanical quantities such as distances and lifetimes and their counterparts in the field of electromagnetism. Over time we have come to believe that we experience electric charge directly, for example, but the fact is that this is only a theoretical quantity that allows us to calculate the Coulomb force between particles at any distance. Force is something very tangible whereas charge is not, despite what our intuition and training tell us. The determination of a **B** field also can only be accomplished by measuring the force between electric charges in relative motion. All the other myriad quantities mentioned in Table 1 have definitions that in one way or another involve explicit measurements of such forces.

It should therefore not be surprising to discover that there is an element of arbitrariness in the way the kinetic and gravitational scaling of electromagnetic quantities can be specified theoretically. As long as one remains consistent, that is, adheres to the simple restrictions outlined in Sect. II, one system of electromagnetic units will inevitably lead to the same set of measured values for force, distance and elapsed time as any other. This realization is at odds with what one commonly finds in the literature (Sard, R. D. 1970b), namely that electric charge is a relativistic invariant. The argument given for the latter conclusion is typically based on the relativity principle and the fact that *in situ* measurements of a given charge are completely independent of the state of motion of the observer. The same can be said of the lifetimes of metastable particles, however, and we know that these quantities do change with the state of motion or position in a gravitational field.

The discovery of time dilation actually requires that one be more specific in defining the relativity principle than is usually the case. One should add that, although the laws of physics are the same in all inertial systems, the *units* in which the various quantities contained in them are expressed are *not* necessarily equal. *In situ* measurements do not vary from one inertial system to another for the simple reason that the scaling of such units is *completely uniform*. The same holds true for *in situ* measurements at different gravitational potentials. The only way that one can discern how such units change is to carry out measurements in which the observer is not at the same gravitational potential as the object or is in relative motion to it. Even when this is done, however, one is still free to choose how electromagnetic quantities should scale because of the redundancy in the definition of electric charge and permittivity discussed in Sect. II. More details concerning this subject may be found in earlier references (Buenker, R. J. 2015; & Buenker, R. J. 2014).

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