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Review Article

Extinction Results for a Quasilinear Parabolic Equation with Nonlinear Source

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Abstract: In this paper, we deal with the extinction of solution of the initial boundary value problem of a quasilinear parabolic equation $u_t = \Delta_p u + \lambda |u|^{q-2} u$ in a bounded domain of R^N. We prove that extinction of the solution.

Keywords: Extinction; quasilinear parabolic equation; nonlinear source.

INTRODUCTION

In this paper, we are interested in the nonnegative solutions for the singular nonlinear diffusion problem

$$\begin{cases} u_t = \operatorname{div}(|\nabla u|^{p-2}\nabla u) + \lambda |u|^{q-2}u & (x,t) \in \Omega \times (0,T), \\ u(x,t) = 0, & (x,t) \in \partial\Omega \times (0,T), \\ u(x,0) = u_0(x), & x \in \Omega, \end{cases} \tag{1.1}$$

Where $T > 0$, Ω is a bounded domain in R^N with appropriately smooth boundary $\partial\Omega$, $1 < p, q$, $\lambda > 0$ and $u_0(x)$ satisfies the following conditions:

$$(H) \qquad 0 \le u_0 \in C(\overline{\Omega}) \cap W^{1,p}(\Omega), \quad u_0 = 0 \ \text{ on } \ \partial\Omega.$$

This type of equations arise in biological and astrophysical context. In combustion theory, for instance, the function $u(x,t)$ represents the temperature; the term $\operatorname{div}(|\nabla u|^{p-2}\nabla u)$ represents the thermal diffusion and $\lambda |u|^{q-2}u$ is a source. Equations (1.1) arises also in some models describing physical phenomenon. For example, when $a = 0$, equation (1.1) is the evolution p-Laplacian. Equations of this form are mathematical models occurring in studies of non-Newtonian fluid theory [3,4], non-Newtonian filtration theory [5] and the turbulent flow of a gas in porous medium [6]. When $p = 2$, the blow-up properties of the semilinear equation (1.1) has been investigated by many researchers. For $p \neq 2$, the main interest in the past twenty years lies in the regularities of weak solutions of the quasilinear parabolic equation (see [8-11]).

In present paper, our interest is to investigate the extinction of the nonnegative solution u in (1.1), i.e. there exists a finite time $T > 0$, such that the solution is nontrivial for $0 < t < T$, but $u(x,t) \equiv 0$ for all $(x,t) \in \Omega \times [T, +\infty)$. In this case, T is called the extinction time. The first result on extinction is due to

Kalashnikov in 1974 (see [1]). For homogeneous Dirichlet boundary value problem of semilinear heat equation as follows:

$$\begin{cases} u_t = \triangle u - u^q, & (x,t) \in \Omega \times (0,\infty), \\ u(x,t) = 0, & (x,t) \in \partial\Omega \times (0,T), \\ u(x,0) = u_0(x), & x \in \Omega, \end{cases} \tag{1.2}$$

The most complete conclusions were obtained in [2]: A nontrivial solution of (1.2) vanishes in finite time if and only if $0 < q < 1$ (i.e. strong absorption will cause extinction in finite time). More extinction properties of (1.2) have also been considered extensively by several authors (e.g. [7, 12] and the references therein).

In [2], Gu also gave a simple statement of the necessary and sufficient conditions of extinction of the solution to the following problem:

$$\begin{cases} u_t = \operatorname{div}(|\nabla u|^{p-2}\nabla u) - \lambda u^q, & (x,t) \in \Omega \times (0,\infty), \\ u(x,t) = 0, & (x,t) \in \partial\Omega \times (0,T), \\ u(x,0) = u_0(x), & x \in \Omega, \end{cases} \tag{1.3}$$

With $\lambda > 0$. He proved that if $p \in (1,2)$ or $q \in (0,1)$ the solutions of the problem vanish in finite time, but if $p \geq 2$ and $q \geq 1$, there is non-extinction. In the absence of absorption (i.e.$\lambda = 0$), Dibenedetto [13] and Yuan *et al.* [14] proved that the necessary and sufficient conditions for the extinction to occur is $p \in (1,2)$.

In [15], Li established conditions of extinction of the solution to the following porous medium problem:

$$\begin{cases} u_t = \Delta u^m + \lambda u^p, & (x,t) \in \Omega \times (0,\infty), \\ u(x,t) = 0, & (x,t) \in \partial\Omega \times (0,T), \\ u(x,0) = u_0(x), & x \in \Omega, \end{cases} \tag{1.4}$$

with $0 < m < 1$, $p, \lambda > 0$. Where Ω is an open bounded domain with smooth boundary? They showed that if $p > m$, the solution with small initial data vanishes in finite time, and if $p < m$, the maximal solution is positive for all $t > 0$. If $p = m$, then first eigenvalue of the Dirichlet problem plays a role.

Roughly speaking, for the problems (1.2) and (1.3), there is a comparison between the diffusion term and the absorption term, and fast diffusion or strong absorption will lead any bounded nonnegative solution to zero in finite time. But in (1.1) and (1.4), the nonlinear source u^{p-1} is physically called the ``hot source", while in (1.2) and (1.3), the source $u^q$ is called the ``cool source"; the different sources have completely different influences on the properties of solutions (we refer the reader to [13, 15, 16]). For problem (1.1), with a ``hot source", it has been shown that the solution blows up in finite time for sufficiently large initial data (see [17]). In this paper we will show that the solution of (1.1) vanishes in finite time for sufficiently small initial data.

It is well known that Eq. (1.1) is degenerate if $p > 2$ or singular if $1 < p < 2$, since the modulus of ellipticity is degenerate ($p > 2$) or blows up ($1 < p < 2$) at points where $\nabla u = 0$, and therefore there is no classical solution in general. For this, a nonnegative weak solution for problem (1.1) is defined as follows.

For convenience, define $\Omega_T = \Omega \times (0,T),\ T > 0$. Denote $\widetilde{\Omega}$ and $\widetilde{\Omega}_\rho$ for $\rho > 0$ by

$$\widetilde{\Omega} = \{x \in \Omega; u_0(x) > 0\}, \qquad \widetilde{\Omega}_\rho = \{x \in \widetilde{\Omega}; \operatorname{dist}(x, \partial\widetilde{\Omega}) > \rho\}.$$

Definition 1.1. A nonnegative function u is called a weak solution of problem (1.1), if and only if $u \in L^\infty(\Omega_T) \cap L^p(0,T;W_0^{1,p}(\Omega)), u_t \in L^2(\Omega_T)$, and there holds

$$\int\int_{\Omega_T} (-u\varphi_t + |\nabla u|^{p-2}\nabla u \nabla\varphi - \lambda |u|^{q-2}u\varphi)dxdt = 0, \tag{1.5}$$

And

$$\lim_{t\to 0^+}\int_\Omega |u(x,t)-u_0(x)|dx=0,$$

Where the text function $\varphi(x,t)\in C_0^\infty(\Omega_T)$.

Remark 1.2. To define weak solutions for the problem with arbitrary nonnegative function $\psi\in L^\infty(\Omega_T)\cap L^p(0,T;W_0^{1,p}(\Omega))$ as its boundary value, it suffices to require instead of $u\in L^\infty(\Omega_T)\cap L^p(0,T;W_0^{1,p}(\Omega))$. Furthermore, because of the denseness of $C_0^\infty(\Omega_T)$ in$L^p(0,T;W_0^{1,p}(\Omega))$, one can assert that the above equality holds for any$\varphi\in L^\infty(\Omega_T)\cap L^p(0,T;W_0^{1,p}(\Omega))$.

Similarly, to define a subsolution (resp., supersolution) $\underline{u}(x,t)$ (resp., $\overline{u}(x,t)$), we need only demand $\underline{u}(x,0)\le u_0(x)$ (resp., $\overline{u}(x,0)\ge u_0(x)$) in Ω, $\underline{u}(x,t)\le 0$ (resp.,$\overline{u}(x,t)\ge 0$) on$\partial\Omega\times[0,T]$, and equality in (1.4) is replaced by $\le$ (resp.,$\ge$) for every $\varphi(x)>0$.

The rest of the paper is organized as follows. In Section 2, we will give some preliminary lemmas. We will prove extinction results in Section 3.

Preliminary

Before studying our problem, we will give some lemmas, which will be useful tools in our later proofs. First, we establish the comparison principle; we begin with a simple lemma which provides the necessary algebraic inequality.

Lemma 2.1. For all $x,y\in R^N$,there holds

$$(|x|^{p-2}x-|y|^{p-2}y)\cdot(x-y)\ge\begin{cases} C_1(|x|+|y|)^{p-2}|x-y|^2, & :\text{ if } \quad p>1,\\ C_2|x-y|^p, & :\text{ if }\quad p>2,\end{cases}$$

Where C_1 and C_2 are positive constants depending only on p.

For the detail of this lemma, we refer the reader to Lemma 4.10 in [17] and Lemma 2.1 in [18]. Now, we prove the following comparison lemma:

Lemma 2.2. Suppose that $\underline{u}(x,t)$, $u(x,t)$ are a subsolution and a supersolution of (1.1) respectively, then $\underline{u}(x,t)\le\overline{u}(x,t)$ a.e. in Ω_T.

Proof. For small δ, set

$$\rho_\delta(\sigma)=\begin{cases}0 & : \quad \delta<0,\\ \frac{\sigma}{\delta} & : \quad 0\le\sigma\le\delta,\\ 1 & : \quad \sigma<\delta,\end{cases}$$

then $\rho_\delta(\sigma)$ is a piecewise differential function.

Letting $\varphi(x,t)=\rho_\delta((\underline{u}-\overline{u})(x,t))$, it is easy to verify that $\varphi(x,t)$ is an admissible function in Definition 1.1. According to (1.5) in Definition 1.1, we have

$$\int_\Omega \underline{u}(x,t_2)\varphi(x,t_2)dx-\int_\Omega \underline{u}(x,t_1)\varphi(x,t_1)dx\le\int_{t_1}^{t_2}\int_\Omega\{\underline{u}\varphi_s-|\nabla\underline{u}|^{p-2}\nabla\underline{u}\nabla\varphi+\lambda|\underline{u}|^{q-2}\underline{u}\varphi\}dxdt, \tag{2.1}$$

And

$$\int_\Omega \overline{u}(x,t_2)\varphi(x,t_2)dx-\int_\Omega \overline{u}(x,t_1)\varphi(x,t_1)dx\le\int_{t_1}^{t_2}\int_\Omega\{\overline{u}\varphi_s-|\nabla\overline{u}|^{p-2}\nabla\overline{u}\nabla\varphi+\lambda|\overline{u}|^{q-2}\overline{u}\varphi\}dxdt. \tag{2.2}$$

Let $t_1=\tau,\ t_2=\tau+h<T,\ \tau,h>0$, and $\omega=\underline{u}-\overline{u}$, then by (2.1) and (2.2), we obtain

$$\int_\Omega\omega(x,\tau+h)\rho_\delta(\omega(x,\tau+h))dx\le\int_\Omega\omega(x,\tau)\rho_\delta(\omega(x,\tau))dx+\int_\tau^{\tau+h}\int_\Omega\omega\rho_\delta'(\omega)\omega_s dxds$$

$$-\int_\tau^{\tau+h}\int_\Omega(|\nabla\underline{u}|^{p-2}\nabla\underline{u}-|\nabla\overline{u}|^{p-2}\nabla\overline{u})\nabla\rho_\delta(\underline{u}-\overline{u})dxds+\lambda\int_\tau^{\tau+h}\int_\Omega\rho_\delta(\omega(x,s))(|\underline{u}|^{q-2}\underline{u}-|\overline{u}|^{q-2}\overline{u})\varphi dxds. \tag{2.3}$$

Dividing (2.3) by h and integrating τ over $(0,t),\ t<T$, we obtains

$$\int_0^t\frac{1}{h}\int_\Omega\omega(x,\tau+h)\rho_\delta(\omega(x,\tau+h))dxd\tau\le\int_0^t\frac{1}{h}\int_\Omega\omega(x,\tau)\rho_\delta(\omega(x,\tau))dxd\tau$$
$$+\int_0^t\frac{1}{h}\int_\tau^{\tau+h}\int_\Omega\omega\rho_\delta'(\omega)\omega_s dxdsd\tau-\int_0^t\frac{1}{h}\int_\tau^{\tau+h}\int_\Omega(|\nabla\underline{u}|^{p-2}\nabla\underline{u}-|\nabla\overline{u}|^{p-2}\nabla\overline{u})\nabla\rho_\delta(\underline{u}-\overline{u})dxdsd\tau$$
$$+\lambda\int_0^t\frac{1}{h}\int_\tau^{\tau+h}\int_\Omega\rho_\delta(\omega(x,s))(|\underline{u}|^{q-2}\underline{u}-|\overline{u}|^{q-2}\overline{u})\varphi dxdsd\tau. \tag{2.4}$$

Let $h\to 0^+$, by the properties of Steklov's averages (see [2,11]) and simple calculations, we get

$$\int_\Omega\omega(x,t)\rho_\delta(\omega(x,t))dx\le\int_\Omega\omega(x,0)\rho_\delta(\omega(x,0))dx+\int_0^t\int_\Omega\omega\rho_\delta'(\omega)\omega_s dxds$$
$$-\int_0^t\int_\Omega(|\nabla\underline{u}|^{p-2}\nabla\underline{u}-|\nabla\overline{u}|^{p-2}\nabla\overline{u})\nabla\rho_\delta(\underline{u}-\overline{u})dxds+\lambda\int_0^t\int_\Omega\rho_\delta(\omega(x,s))(|\underline{u}|^{q-2}\underline{u}-|\overline{u}|^{q-2}\overline{u})\varphi dxds. \tag{2.5}$$

Now, we deal with the terms in (2.5). Firstly, define

$$I_1=\{(x,s)\in\Omega_t\ :\ \underline{u}(x,s)=0\};$$
$$I_2=\{(x,s)\in\Omega_t\ :\ \underline{u}(x,s)\neq 0\quad\text{and}\quad\overline{u}(x,s)=0\};$$
$$I_3=\{(x,s)\in\omega_t\ :\ \underline{u}(x,s)\neq 0\quad\text{and}\quad\overline{u}(x,s)\neq 0\}.$$

Since $\rho_\delta(\omega(x,s))\equiv 0$ in I_1, $\omega_+(x,s)=\underline{u}(x,s)$ in I_2, and $\underline{u},\overline{u}\in L^\infty(\Omega_T)$, we have

$$\int_0^t\int_\Omega\rho_\delta(\omega(x,s))(|\underline{u}|^{q-2}\underline{u}-|\overline{u}|^{q-2}\overline{u})dxds\le(\int\int_{I_1}+\int\int_{I_2}+\int\int_{I_3})(|\underline{u}|^{q-2}\underline{u}-|\overline{u}|^{q-2}\overline{u})\rho_\delta(\omega(x,s))dxds$$
$$\le 0+\int\int_{I_2}|\underline{u}|^{q-2}\underline{u}\omega_+dxds+\int\int_{I_3}(|\underline{u}|^{q-2}\underline{u}-|\overline{u}|^{q-2}\overline{u})\omega_+dxds\le M\int_0^t\int_\Omega\omega_+dxds,$$

And

$$|\int_0^t\int_\Omega\omega\rho_\delta'(\omega)\omega_s dxds|=|\int_\Omega\int_{\omega(0)}^{\omega(t)}\omega\rho_\delta'(\omega)d\omega dx|\le\int_\Omega\int_{\omega(0)}^{\omega(t)}\omega_+|\rho_\delta'(\omega)|d\omega dx$$
$$\le\frac{1}{\delta}\int_\Omega\int_0^\delta\omega_+d\omega dx\to 0,\qquad\text{as}\quad\delta\to 0^+.$$

Secondly, for $p>1$, by Lemma 2.1, we have

$$(|\nabla\underline{u}|^{p-2}\nabla\underline{u}-|\nabla\overline{u}|^{p-2}\nabla\overline{u})\nabla\rho_\delta(\underline{u}-\overline{u})\ge 0.$$

Finally, we have $\int_\Omega\omega(x,0)\rho_\delta(\omega(x,0))dx\equiv 0$, and $\rho_\delta'\ge 0$ a.e. in R, $\omega\rho_\delta(\omega(x,t))$ increasing and tends to ω_+ as $\delta^+\to 0$.

Hence, we may let $\delta^+\to 0$ in (2.5) to obtain

$$\int_\Omega\omega_+(x,t)dx\le\lambda M\int_0^t\int_\Omega\omega_+(x,t)dxds.$$

By Gronwall's inequality, we obtain $\int_\Omega\omega_+(x,t)dx\equiv 0$, i.e. $u\le\overline{u}$ a.e. in Ω_T. The proof is complete.

The first eigenvalue λ_1 of the following problem plays a crucial role:

$$-\mathrm{div}(|\nabla\phi|^{p-2}\nabla\phi)=\lambda|\phi|^{p-2}\phi,\quad\text{in}\ \Omega;\quad\phi|_{\partial\Omega}=0. \tag{2.6}$$

Next we will introduce the following lemma on the properties of the first eigenvalue λ_1 and the corresponding eigenfunction $\phi(x)$.

Lemma 2.3. There exists a positive constant $\lambda_1(\Omega)$ with the following properties:

(i) For any $\lambda < \lambda_1(\Omega)$, the eigenvalue function (2.6) has only the trivial solution $\phi(x) \equiv 0$.

(ii) There exists a positive solution $\phi \in W_0^{1,p}(\Omega) \cap C(\overline{\Omega})$ of (2.6) if and only if $\lambda = \lambda_1(\Omega)$.

(iii) The collection consisting of all solutions of (2.6) with $\lambda = \lambda_1(\omega)$ is a one-dimensional vector space.

(iv) If Ω_1 and Ω_2 are bounded domains with smooth boundary satisfying $\Omega_1 \subset \Omega_2$, then $\lambda_1(\Omega_1) > \lambda_1(\Omega_2)$.

(v) Let $\{\Omega_n\}$ be a sequence of bounded domains with smooth boundaries such that $\Omega_n \subset \Omega_{n+1}$ and $\cup_{n=1}^{\infty}\Omega_n = \Omega$, then $lim_{n\to\infty}\lambda_1(\Omega_n) = \lambda_1(\Omega)$.

This lemma follows from Lemmas 2.1, 2.2 in [19] and Lemma 1.1 in [17].

The properties of the first eigenvalue θ_1 and the corresponding eigenfunction $\psi(x)$ of the eigenvalue problem

$$\triangle\psi = \theta\psi, \quad \text{in} \quad \Omega; \qquad \psi|_{\partial\Omega} = 0, \tag{2.7}$$

Are well known (see [20]). Moreover, we can define θ_1 using the “Rayleigh quotient”:

$$\theta_1 = \inf_{u\in H_0^1(\Omega), u\neq 0} \frac{\int_\Omega |\nabla u|^2 dx}{\int_\Omega u^2 dx}. \tag{2.8}$$

We will give a similar quotient for the first eigenvalue λ_1 of (2.6) as follows.

Extinction of the solution

In this section, we consider the extinction of the solution to problem (1.1).

Theorem 3.1. Let u be a weak solution of (1.1), then for sufficiently small initial data, there exists a finite time T such that $u \equiv 0$ for all $(x, t) \in \overline{\Omega} \times (T, +\infty)$.

Proof. Multiplying the first equation of (1.1) by u^s, $s > 0$, and integrating over Ω, we obtain

$$\frac{1}{s+1}\frac{d}{dt}\int_\Omega u^{s+1}dx + \frac{sp^p}{(s+p-1)^p}\int_\Omega |\nabla u^{\frac{p+s-1}{p}}|^p dx = \lambda\int_\Omega |u|^{q-2}u^{s+1}dx. \tag{3.1}$$

By Lemma 2.2, if $u_0 \le k\phi(x)$ in Ω, for sufficiently small $k > 0$, $\phi(x)$ is the positive first eigenfunction of (2.6), and $\max_{x\in\Omega}\phi(x) = 1$, it can be easily verified that $k\phi(x)$ is a supersolution of (1.1); then, we have $u(x,t) < k\phi(x)$ for all $(x,t) \in \Omega \times (0,1)$. From this, (3.1) can be rewritten as

$$\frac{1}{s+1}\frac{d}{dt}\int_\Omega u^{s+1}dx + \frac{sp^p}{(s+p-1)^p}\int_\Omega |\nabla u^{\frac{p+s-1}{p}}|^p dx \le k^{q-2}\int_\Omega u^{s+1}dx.$$

Then we have

$$\frac{1}{s+1}\frac{d}{dt}\int_\Omega u^{s+1}dx \le k^{q-2}\int_\Omega u^{s+1}dx.$$

By integration, we have

$$\int_\Omega u^{s+1}dx \le (s+1)k^{q-2}t\int_\Omega u_0^{s+1}dx,$$

To which the above argument can be applied. The proof of Theorem 3.1 is complete.

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