East African Scholars Journal of Engineering and Computer Sciences
Abbreviated Key Title: East African Scholars J Eng Comput Sci
ISSN: 2617-4480 (Print) & ISSN: 2663-0346 (Online)
Published By East African Scholars Publisher, Kenya

OPEN ACCESS

Volume-5 | Issue-5 | Sept-2022 | DOI: 10.36349/easjecs.2022.v05i05.003

Review Article

Mathematical Modeling of the Effect of Cut-Off in Controlling Seepage beneath Hydraulic Structures

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Article History
Received: 25.08.2022
Accepted: 22.09.2022
Published: 27.09.2022

Journal homepage:
https://www.easpublisher.com

Quick Response Code

Abstract: Seepage is among the major causes of damage to dams and other hydraulic structures. Studying the hydraulic gradient and drift beneath hydraulic structures give a general idea about their safety. The objective of the paper is to develop a mathematical model of the effect of cut-off in controlling seepage beneath dams and other hydraulic structures. Analytical method was applied in the solution of the derived differential equation. The model exhibited a fairly constant reduction in the drift from upstream to downstream. A comparison of model shows that it compared favourably with the existing model with the advantage of being more simplified and easy to apply.
Keywords: Analytical Method, Cut-off Wall, Drift, Hydraulic Gradient, Hydraulic Structures, Modeling, Seepage.

1.0 INTRODUCTION

Seepage can be defined as the flow of groundwater by gravity in a permeable soil. Subsurface flow proceeds from a region of high head to that of a lower head. Seepage is among the major causes of failure in hydraulic structures such as dams [1]. Adequate attention is required in monitoring the hydraulic gradient and drifts in the foundation of hydraulic structures so as to control and eliminate potential damages such as piping and erosion of foundation soil.

Mohamad *et al.*, [2] predicted the effect of cut-off wall on the drift under the pressure of diversion dam. They recorded the minimum uplift pressure when the cut-off wall was located at the upstream (heel) of the dam.

Alghazali and Alnealy [3] studies the effect of the cutoff inclination angle on uplift pressure, quantity of seepage and factor of safety and inferred that downstream cutoff inclined towards the downstream side with θ equal 120^0 is beneficial in increasing the safety factor against piping. Upstream cutoff inclined towards the downstream side with θ equal 45^0 is beneficial in reducing uplift pressure and quantity of seepage.

Abdoreza *et al.*, [4] studied the effect of cutoff wall conditions on the seepage characteristics. They found that the optimum position of the cut off wall is about 0.4 – 0.6 of the width of the dam from the heel. The best options of cutoff wall permeability for reduction in seepage characteristics are between 10^{-8} to 10^{-9} m/s.

The objective of the paper is to develop a mathematical model of the effect of cutoff in controlling seepage beneath dams and other hydraulic structures.

2.0 MODEL DEVELOPMENT

Cutoff has substantial influence on the characteristics of seepage field, considering it obvious effect for blocking groundwater seepage. It changes seepage field form and reduce the seepage losses by forming a barrier. The cutoff minimizes seepage flow in the longitudinal direction by creating a hydraulic gradient between the faces of the cutoff wall.

2.1 VARIABLES AND PARAMETERS

The independent variables are the longitudinal and transverse directions of seepage flow (ie x and y directions). The dependent variable is the field variable known as the piezometric head (h).

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The parameters consist of the following. Hydraulic gradient (i), permeability of the foundation soil (K), infiltration, evaporation and permeability of the cutoff wall.

2.2 DERIVATION OF THE GOVERNING EQUATION

Consider at two dimensional seepage flow of water through an element of soil of size dx, dy and dz as showing Fig 1. The third dimension (z-axis) is large and can be taken as unity.

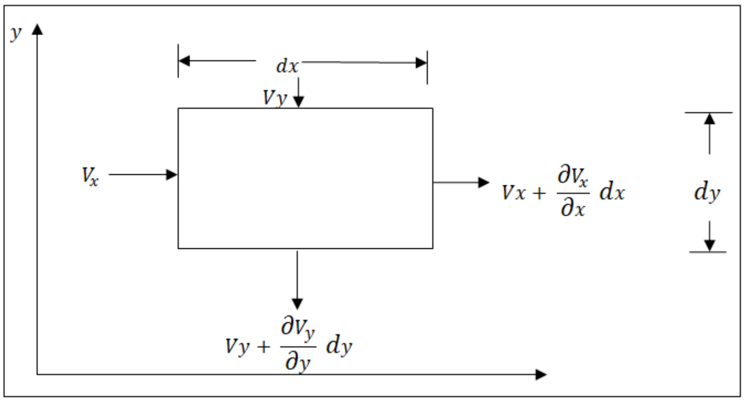

Fig 1: Two Dimensional flow

As the flow is ready, the discharge entering the element is equal to that leaving the element.

$$V_x\, \partial_y + V_y d_x = \left(Vx + \frac{\partial V_x}{\partial x}.dx\right) dy + \left(Vy + \frac{\partial v_y}{\partial y}.dy\right) dx$$

$$\left(\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y}\right) dxdy = 0 \text{ .... (1)}$$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0 \text{ .... (2)}$$

Equation (2) is the continuity equation for two dimensional flow. The longitudinal and transverse components of the hydraulic gradient are respectively.

$$i_x = -\frac{\partial h}{\partial x} \text{ and } i_y = -\frac{\partial h}{\partial y} \text{ .... (3)}$$

The minus sign indicates that the head decreases in the direction of flow

From Darcy's law

$$V_x = -K_x \frac{\partial h}{\partial x}, V_y = -K_y \frac{\partial h}{\partial y} \text{ .... (4)}$$

Substituting to equation (2) it becomes

$$-K_x \frac{\partial^2 h}{\partial x^2} - K_y \frac{\partial^2 h}{\partial y^2} = 0 \text{ .... (5)}$$

The foundation soil is assumed to be isotropic

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \text{ .... (6)}$$

The cutoff wall will create hydraulic gradient in the seepage field in the longitudinal direction of flow. Equation (7) integrates the effect of the cutoff wall thins.

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial h}{\partial x} = 0 \text{ .... (7)}$$

The cutoff wall is also assumed to have effect only on the longitudinal direction of seepage flow (i.e. x – direction), therefore the transverse component of the seepage flow can be ignored.

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial h}{\partial x} = 0 \text{ .... (8)}$$

2.3 MODEL SOLUTION

Equation (8) is a second order ordinary differential equation (Laplace equation).

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial h}{\partial x} = 0$$

Boundary conditions.

$$h(x = 0) = h_1 \text{ .... (9)}$$

$$h(x = l) = h_2 \text{ .... (10)}$$

The characteristic equation is

$$m^2 + m = 0$$

$$m\,(m + 1) = 0$$

$$m = 0 \text{ or } m = -1$$

Hence the general solution becomes

$$h\,(x) = A \cos(-x) + B \sin(-x) \text{ .... (11)}$$

Applying the boundary conditions

$$h(0) = A \text{ Cos } 0 + B \text{ Sin } 0 = h_1$$

$$A = h_1 \text{ .... (12)}$$

$$h(l) = h_1 \text{Cos}(-l) + B\text{sin}(-l) = h_2$$

$$B = \frac{h_2}{\text{Sin}\,(-l)} - \frac{h_1 \text{ Cos}\,(-l)}{\text{Sin}(-l)} \text{ .... (13)}$$

Substituting equations (12) and (13) to (11)

$$h\,(x) = h_1 \text{ Cos}(-x) + h_2 \left[\frac{\text{Sin}(-x)}{\text{Sin}\,(-l)}\right] - \frac{h_1 \text{ Cos}\,(-l)\text{Sin}(-x)}{\text{Sin}(-l)} \text{ .... (14)}$$

3.0 MODEL VERIFICATION

The analytical model developed was verified by comparing the results with that of [2], which is the usual seepage equation ($\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$) . The experimental data was taken from a diversion dam with upstream water level 20 meters and downstream water level 10 meters, modeled 16 meters on foundation soil 10 meters thickness. The dam comprises a cut-off wall of 10 meters depth located at the upstream (heel) of the dam.

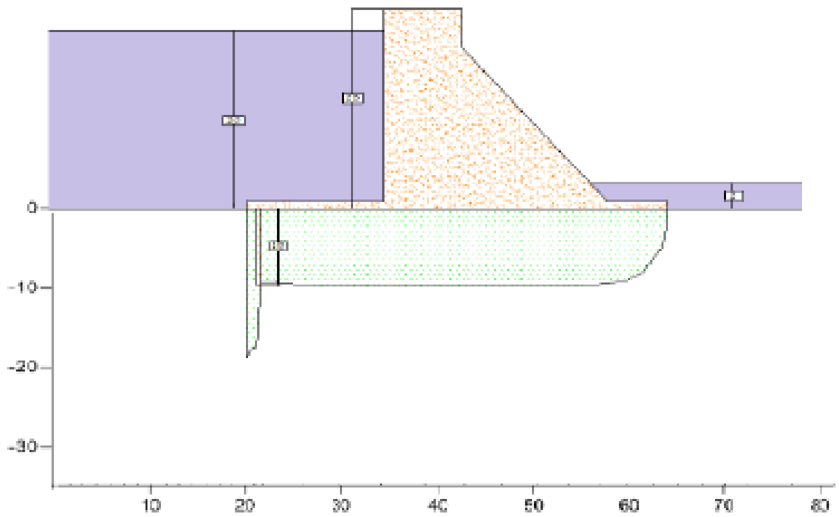

Fig 2: The diversion dam [2]

A comparison of the two models in figure 3 shows that the present model adequately predicted the drift under the diversion dam. The Mohamad et al model exhibited a reduced drift at the upstream and a higher drift at the downstream in relation to the present model. On the other hand, the present model exhibited a fairly constant reduction in the drift from upstream to downstream. Generally the present model is adequate for the prediction of the drift under the pressure of a dam or any hydraulic structure.

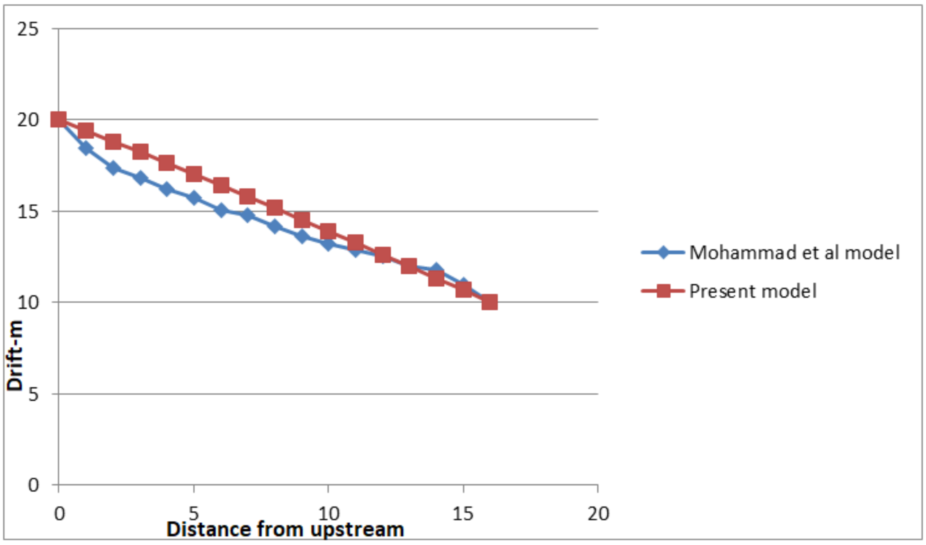

Fig 3: Comparison of the two models under pressure at Diversion Dam

4.0 CONCLUSION

Ease of use, accuracy and applicability are among the attributes that determine the success of a mathematical model. The analytical model developed was successfully applied in the prediction of the drift beneath a diversion dam. The model compared favourably with the existing model with the advantage of being more simplified and easy to use. It is therefore suitable for use in predicting the effect of cut-off wall on drift beneath hydraulic structures.

REFERENCES

1. Arora, K. R. (2008). "Soil Mechanics and Foundation Engineering" 7th Edition, Standard Publishers Distributors, 1705-B, Nai Sarak, 1066 Delhi-110006, p.163-164. ISBN: 81-8014.
2. Mohamad, H. H. N. B., Sajjad, S, Amir, H. H. A., & Hassan, T. P. (2014). "Simulation of Flow From Under the Diversion Dam and Cut-off Wall Effects with Finite Volume Method" *Applied Mathematics in Engineering, Management and Technology*, The

First National e-conference on Advances in Basic Sciences and Engineering, p. 136-144. www.amiemt-journal.com.
3. Alghazali, N. O. S., & Alnealy, H. K. T. (2015). “Experimental Analysis of Seepage in Soil Beneath a Model of a Gravity Dam” *American Journal of Civil Engineering*, 3(3), 64-69. ISSN: 2330-8729 (Print); ISSN: 2330-8737 (Online).
4. Abdoreza, A. S., Hanieh, A., Seyad, P. N., & Ali, K. (2014). “The Effect of the Cutoff Wall Conditions on the Seepage Characteristics of Homogeneous Earth-fill Dams Using SEEP/W” *WALIA JOURNALS*, 30(S2), 176-182. ISSN: 1026-3861.

Cite This Article: Ugwuanyi, D. C, Enem, J. I, Ugwu, J. N (2022). Mathematical Modeling of the Effect of Cut-Off in Controlling Seepage beneath Hydraulic Structures. *East African Scholars J Eng Comput Sci*, *5*(5), 77-80.