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Review Article

Zero Points and Their Distributions of Real and Complex Functions

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Abstract: The zero point of function is an important feature of function, and it is an important medium for communication between function and equation. This paper analyzes and summarizes the existence, number and distribution of zero points of real and complex functions, and illustrates them with examples, aiming at deepening readers' understanding of zero points of functions and playing a certain reference role for teachers' teaching and students' learning.
Keywords: Real function; Complex function; Entire function; twelve o'clock at night.

1. INTRODUCTION

As we all know, the main research object of Advanced Mathematics is the function defined on the real number set, and the main research object of Complex Variable Function is the function defined on the complex number field. These two courses mainly introduce the limit, continuity, derivative, integral, function term series, etc. of functions, and they have important applications in mathematics, physics, electrical engineering, communication and other disciplines. For the convenience of narration, this paper uses x A variable that represents real, using z A variable that represents a complex.

To understand the form or nature of a real function or a complex function, the zero point of a function is particularly important. The so-called zero point of a function refers to the point where the value of the function is zero. Sometimes, when we know the zero point of a function, we can determine the form of the function. For example, we know that the three zero points of a unary real cubic function are 1, 2, 3, then the form of this function must be $f(x)=a(x-1)(x-2)(x-3)$, in which a Is a non-zero real constant. In addition, the zero point of the function is closely related to the root of the equation, such as quadratic equation with one variable. $ax^2+bx+c=0$ (a,b,c Is the root of the real number), which can be converted into a quadratic function of one variable. $f(x)=ax^2+bx+c$ The problem of zero point.

Because the negative number can't be opened in the real number field, there are some limitations in studying the zero of the function in the real number field, such as $f(x)=x^2+1$ No real zero. But the imaginary number is introduced. i ($i^2=-1$), the complex function $f(z)=z^2+1$ There are two zeros in the complex field. i and $-i$ The introduction of complex numbers is an expansion of number field and a revolution in the history of mathematics, which is of great significance.

2. THE EXISTENCE AND DISTRIBUTION OF FUNCTION ZEROS

On the existence of zero point of continuous function in closed interval, a basic and important result is as follows:

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Theorem 1 [1] Ruo function $f(x)$ In a closed interval $[a,b]$ Continuous, and $f(a)\cdot f(b)<0$, there is at least one point $x_0 \in (a,b)$, make $f(x_0)=0$.

This theorem is generally called the zero point theorem or the existence theorem of roots, which is equivalent to the following intermediate value theorem.

Theorem 2 [1] Set function $f(x)$ In a closed interval $[a,b]$ Continuous, and $f(a)\neq f(b)$, which is between $f(a)$ and $f(b)$ Any real number between μ ($f(a)<\mu<f(b)$ or $f(b)<\mu<f(a)$), there is at least a little $x_0 \in (a,b)$, make $f(x_0)=\mu$.

Make function $f(x_0)=\mu$ Point x_0, we usually become $f(x)$ taxi μ Point. Theorem 1 and Theorem 2 respectively give the sum of zero points of continuous functions on closed interval. μ Sufficient conditions for the existence of a point. Although the conditions and conclusions of the theorem are simple and clear, it is not easy to prove the theorem. It needs to use the completeness theory of real numbers, for example, it can be proved by bounded theorem or interval sleeve theorem, which can be seen in [1].

The zero point theorem not only solves the problem of the existence of the zero point of a function, but actually gives the distribution range of the zero point. For the existence and distribution of the zero point of a complex analytic function, there is the following famous Rouche theorem.

Theorem 3 [2] set C It's a weekly line, a function $f(z)$ and $g(z)$ Meet the conditions:

(1) $f(z)$ and $g(z)$ at C Are resolved internally, and continue to C ;

(2) at C Go, $|f(z)|>|g(z)|$,

Then function $f(z)$ and $f(z)+g(z)$ at C There are as many zeros (counting numbers) inside.

Example proof: function $f(z)=z^5-z^3+5$ The zero point is all there. $1<|z|<2$ Inside.

Prove: order $g(z)=5$, $h(z)=z^5-z^3$, because in the circle $|z|=1$ Yes, there is.

$$|g(z)|=5>2\geq|z^5-z^3|=|h(z)|,$$

So it is known from Theorem 3: $f(z)=g(z)+h(z)$ at $|z|<1$ Neihe $g(z)=5$ at $|z|<1$ As many as zero points. So $f(z)$ at $|z|<1$ There is no internal zero point. On the other hand, because in the circumference $|z|=2$ Yes, there is.

$$|5-z^3|\leq 5+|z|^3=5+8=13<32=2^5=|z^5|,$$

So it is known from Theorem 3: $f(z)=z^5-z^3+5$ All seven zeros of are in $|z|<2$ Inside.

To sum up, the function $f(z)=z^5-z^3+5$ The zero point is all there. $1<|z|<2$ Inside.

In fact, the existence of zero point of real polynomial function can be reduced to seeking one yuan. n Subequation $a_n x^n + a_{n-1}x^{n-1}+\cdots+a_0=0$ The problem of roots. Whether the equation has roots, whether there is a formula solution, and how many roots are, used to be the central problem of algebra. A large number of mathematicians such as Leibniz, Lagrange, Abel, Galois, etc. have made important contributions. If we consider one variable in the complex number field n The existence and number of zeros of polynomial have the following famous basic theorems of algebra.

Theorem 4 [2] On the complex plane, n Subpolynomial

$$f(z)=a_n z^n + a_{n-1}z^{n-1}+\cdots+a_0 \quad (a_n\neq 0)$$

There is at least one zero point.

Theorem 4 can also be described as

Theorem 5 On the complex plane, n Subpolynomial

$$f(z)=a_n z^n + a_{n-1}z^{n-1}+\cdots+a_0 \quad (a_n\neq 0)$$

Qiayou n Zero points (counting).

There are many ways to prove the basic theorem of algebra, and the common way to prove it is to use Liuwei Theorem or Rouche Theorem. See [2] for details.

Theoretically, the degree of complex polynomial function n The larger, the more zeros of the polynomial, so a natural question is if n go ∞, does the complex function have infinite zeros? It is easy to cite counterexamples to show that this proposition is not true, such as complex exponential function. $e^z=1+z+\frac{z^2}{2!}+\cdots+\frac{z^n}{n!}+\cdots$ It has no zero point in the whole complex number field. Does that mean that there is something wrong with our intuition? Actually, we can easily calculate the function. $f(z)=e^z-a$ (a Any non-zero complex number) has an infinite number of zeros. People call analytic functions on the whole complex plane integral functions, and non-polynomial integral functions are usually called transcendental integral functions. The coefficients of the power series expansion of transcendental integral functions have infinite nonzero terms. For the existence and number of zeros of transcendental integral functions, there are the following famous Picard theorems.

Theorem 6 [3] Any transcendental whole function can take any value infinitely many times in complex field, with at most one exception.

This theorem profoundly reveals the existence and number of zero points of transcendental whole functions, which is of great significance. It is not easy to prove this theorem by elementary methods, but it is much easier to prove it if we use the knowledge of value distribution theory.

People have been studying the existence and distribution of function zeros. For example, Riemann conjecture, which is recognized as the most important mathematical conjecture in the field of mathematics today, is about the distribution of function zeros. As early as 1859, Riemann, a famous German mathematician, guessed that functions $\varsigma(s)=\sum_{n=1}^{\infty}\frac{1}{n^s}$ All nontrivial zeros of lie on the complex plane. $\operatorname{Re} s=\frac{1}{2}$ Although this conjecture has not been proved, there are more than 1000 mathematical propositions in today's mathematical literature on the premise of the establishment of Riemann conjecture (or its extended form), and there are countless connections between this conjecture and many mathematical branches.

3. APPLICATION OF FUNCTION ZERO THEOREM

In our daily life, we will encounter such a phenomenon. If a chair is placed on an uneven ground, the chair usually has only three feet on the ground. At this time, the chair is unstable, but we can stabilize the chair only by moving it or rotating it a few times. This seemingly unrelated phenomenon can be explained by the zero theorem. For the convenience of description, we make the following assumptions: (1) The four legs of the chair are as long as each other, and the connection between the four legs is square, ([4] As shown in Figure 1, remember the four legs of the chair.

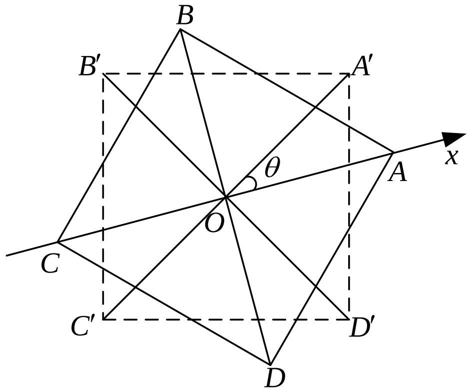

Figure 1

Sequentially A,B,C,D , and A,C and B,D The intersection of the lines is noted as O, around the chair O Point rotation angle x Back, square $ABCD$ Rotate to $A'B'C'D'$. Set A,C The sum of the distances between two feet and the ground is $f(x)$, B,D The sum of the distances between two feet and the ground is $g(x)$, known from hypothesis (2), $f(x)$ and $g(x)$ Are continuous functions. As known from hypothesis (3), for any x , $f(x)$ and $g(x)$ At least one of them is zero. When $x=0$ When, may wish to set $f(0)=0$, $g(0)>0$ In this way, we reduce the problem to the following mathematical problems: known function $f(x)$ and $g(x)$ at $[0,2\pi]$ Continuous, and for arbitrary x , $f(x)\cdot g(x)=0$, and $f(0)=0$, $g(0)>0$, there is at least one point. x_0 , make $f(x_0)=g(x_0)=0$.

Prove: When $x=\frac{\pi}{2}$ When, there is $f(\frac{\pi}{2})>0$ and $g(\frac{\pi}{2})=0$. order $h(x)=f(x)-g(x)$, then $h(x)$ at $[0,2\pi]$ Again, continuously $h(0)=f(0)-g(0)<0$, $h(\frac{\pi}{2})=f(\frac{\pi}{2})-g(\frac{\pi}{2})>0$ So from the zero point theorem, there is at least one point $x_0\in(0,\frac{\pi}{2})$, make $h(x_0)=0$, i.e $f(x_0)=g(x_0)$. due to $f(x_0)\cdot g(x_0)=0$, therefore $f(x_0)=g(x_0)=0$ The proposition is proved

4. SUMMARY

In the 20th century, the theory of meromorphic distribution has been known as one of the most important aspects of the theory of meromorphic distribution. This theory has important applications in many branches of complex analysis, such as complex differential equations, complex dynamical systems and normal family theory

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