

## Review Article

## Pólya's Three Learning Principles and Maths Teaching

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**Abstract:** Pólya thinks the major goal of teaching is to teach students how to ponder. It means that we need not only to give them the knowledge but also to help them improve their ability of using the knowledge and to help them form a good habit of thinking and a good mental state. He also thinks that what the teacher says in class is very important and what is more important is to learn what the students are thinking about. From this standpoint, he presents the three Learning Principles, which is also known as 3 Teaching Principles. These principles are quite practical and are worth practicing and improving in teaching. This article introduces how to use the principles, taking Interpolation Method for example.

**Keywords:** Pólya; Three Learning Principles; Interpolation Method.

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### INTRODUCTION

George Pólya (1887-1985) was an outstanding mathematician and mathematics educator. In 1912, he received his doctorate from Budapest University in Hungary. In 1914, he entered the Swiss Federal Institute of technology in Zurich and moved to the United States in 1940. Since 1942, he has been a professor at Stanford University. Professor Polya is very enthusiastic about mathematics education and pays special attention to the cultivation of mathematical thinking and problem-solving ability. His mathematics and conjecture, how to solve problems, mathematical discovery and other works have been widely praised by readers all over the world and become classic works of mathematics education. Among them, the theme of mathematics and conjecture is the research on the principle and application mode of induction and analogy, The central content of how to solve problems is the research on the general model of problem solving, and the discovery of Mathematics The focus of these books is on the understanding, research and teaching of problem solving. These books talk about the past and the present, have rich and colorful contents, and make people feel fascinating, interesting and unique. In the process of repeated learning, readers can understand how to ask, think, analyze and solve problems, stimulate their internal initiative and creative spirit, so as to make them more attractive Up to now, these

contents still have important enlightening and guiding significance for our mathematics workers and teachers.

#### 1. Pólya's "Three Learning Principles"

One of the most confusing and fascinating links in learning and studying mathematics is how to find and prove theorems. Polya advocates starting from the simplest, then using "reasonable reasoning" methods such as induction and analogy to guess the result, and then using special case test to support or overturn the guess. If the special case test makes us sure that the guess result is correct, we can start to look for strict mathematical proof. On this basis, we can also generalize or specialize the proposition, and use induction and analogy to find more general conclusions or more targeted results. The first step in learning is imitation, and imitation needs an example, which is a model or model. Therefore, Polya especially emphasizes the establishment and application of model.

Polya believes that the fundamental goal of teaching is to "Teach Thinking", that is, not only impart knowledge to students, but also develop students' ability to use what they have learned, so as to help students develop good thinking habits and beneficial mentality. He also believes that what teachers say in class is of course very important, but it is more important to understand what students think. Based on this view, he

put forward "three principles of learning" and "three principles of teaching" [1]:

- **Active learning**

Learning should be proactive. It is difficult to learn anything by reading or listening to classes without using your own brain. Therefore, in order to study effectively, students should find as many materials as possible under given conditions.

From the perspective of teachers, students can actively contribute to the clear expression of the problems they must solve in the future. If students have played a role in the process of putting forward problems, they will be more active in the learning process in the future.

- **Best motivation**

If a student has no learning motivation, he will not take the initiative to learn. He must have some form of motivation. Interest is the best learning stimulus, and the pleasure brought by strong mental activities is the best reward for this activity. Therefore, in order to study effectively, students should be interested in the materials and have fun in learning activities.

From the perspective of teachers, the choice of examples is very important. If possible, it is best related to daily life, and it is best to bring some humor and paradox. Another way is to let the student guess the result or part of the result before he starts solving the problem. Students who have expressed their opinions will be eager to know whether their guess is right, so they will pay more attention to their study.

- **Stage sequence**

Conventional problems are necessary for basic training, but at the appropriate teaching stage, we should introduce some challenging problems, some problems with rich background and worthy of in-depth exploration. Therefore, for effective learning, there should be an exploration stage before the stage of language expression and concept establishment. Finally, the learning income will be transformed into students' talents and character, integrated into it, and become a part of spiritual quality.

For teachers, if there is a suitable and extraordinary problem, students can do some exploration work in advance, which can improve their interest in formal problem-solving, and make some retrospective summary and refinement of the completed solutions, which will be beneficial to the solution of problems in the future.

These three principles have a certain degree of operability and are worthy of our teachers' application and development in teaching practice. Next, we take "interpolation" as an example to illustrate the application of these three principles.

## 2. Teaching points of "interpolation method"

It should not be a problem for students to learn the above contents with interest, but teachers' efforts are needed to make students have further learning motivation. One way is to tell students that interpolation is "useful". You might as well find a problem. The calculation needs to use the "mathematical table". The relevant data do not appear in the table, but the data close to it have results. At this time, the simplest and effective way is to use the two adjacent data for linear interpolation, or use the three adjacent data for quadratic interpolation. The approximate calculation of definite integral (such as calculating area and volume) is also a good application example of interpolation method. Since the method is useful, it is worth our effort to learn it well.

For a mathematical problem, we have some basic problems to consider, such as:

- (I) Is the solution of the interpolation problem unique? How to estimate the error of interpolation approximation? For all mathematical approximation problems, we must think about and answer such questions. It is easy to prove that for the polynomial interpolation problem, the solution is unique, and the error can be expressed by the higher derivative of the function.
- (II) The higher the degree of approximation polynomial, the better? In many cases, the answer is No. The key is that we should have some understanding of the properties of the function, for example, whether it changes continuously or has a breakpoint? Is it a steady change or a drastic change? If there are discontinuities or intervals with drastic changes, the general discussion does not play a great role and needs to be discussed separately. In other cases, it is also possessed by most practical problems, that is, the local part of the function changes smoothly, the local part of the function can be approximated by low-order polynomials, and then the local low-order polynomials can be assembled into a large-scale approximation function according to certain rules.

With the above understanding, we can distinguish two cases for further thinking: (1) regard interpolation as a pure mathematical problem and explore the general interpolation formula. (2) The large-scale interpolation problem is transformed into multiple local interpolation problems, and the local interpolation is determined by low-order polynomial, which is the research content of spline function approximation.

Some problems are selected as pre exploratory problems, which are solved by students independently. In this process, I learned the basic contents and main conclusions of interpolation method, which are

knowledge. In addition, I used the ideas of decomposition and combination and analogy to solve the problems. These belong to the thinking method. Another point is to pay attention to the actual background of the problems, which leads to one problem after another. This is the basic ability that autonomous learning must have. Once these problems are solved, the basic goal of "interpolation" teaching will be achieved. As for spline approximation, it is the comprehensive application of these ideas and methods.

### 3. SUMMARY

Polya's three teaching principles are actually consistent with what we often say "teach students according to their aptitude, explain in simple terms and draw inferences from one instance". However, Polya emphasizes students' subjective initiative and creativity. Only through students' in-depth thinking and independent discovery can we give full play to the maximum teaching benefits.

How should teachers guide students to learn actively? This is a difficult question to answer, but it is a basic question that we must think about every day. Of course, we can use interesting stories to attract students' temporary attention, but the stories do not include all the mathematical content. They are usually just spices

to help us understand complex mathematical theories. After the story, the main driving force that can keep students interested and continue to study is that the learned content is "useful", that is, the learned content can either enable us to better understand society and nature, or enable us to obtain practical benefits. Another important aspect is to let students fully participate in all links of teaching activities, make them actively think and put forward their own views, and improve learning efficiency in the fun of active participation and discovery.

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