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Research Article

Schumann Resonances on Titan

Frederick J. Young, Ph. D
Retired Carnegie Mellon Professor of Applied Space Science, 800 Minard Run Road, Bradford, PA 16701

*Corresponding Author
Frederick J. Young

Abstract: There have been many papers on Schumann resonances on various heavenly bodies. None of them gives the computer programs that calculate the resonances. They leave the interested researcher not knowing how to repeat or verify their results. In this paper a complete finite element program is presented that can be obtained free. Schumann resonances due to variations in the relative permittivity and the ellipticity of Titan are presented.
Keywords: Schumann resonances, ellipticity.

Part I
INTRODUCTION

The Titan-atmosphere system comprises two concentric spheres. The inner sphere is to the first approximation a solid sphere of 2574.73 km radius The atmosphere forms the outer spherical lamina of about 100 km thickness. It contains nitrogen, methane ethane and various other petroleum gases found on Earth. The permittivity of the atmosphere, the ellipticity and the thickness of the spherical lamina are varied to yield the resulting variation caused in the Schumann resonant frequencies. As more space probe measurements become available the program given here can be used to get updated results. The atmosphere of Titan is not expected to be very good electrical conductor. The electric fields and magnetic intensity existing in the atmosphere and the ice layer surrounding the planet are described Maxwell's curl equations as follows:

$$curl(E) = -j\omega\mu_0 H \quad (1)$$
$$curl(H) = j\omega \bullet \epsilon_0 E \quad (2)$$

where H is the magnetic Intensity, E the electric field intensity, ω the frequency, μ_0 the magnetic permeability of space, ϵ_0 the electric permittivity of space and $\bullet$ the relative dielectric constant. Taking the curl of Eq. 2 and substituting $curl(E)$ from Eq. 1 yields

$$curl(curl(H)) = \omega \bullet \epsilon_0 \omega \mu_0 H \quad (3)$$

to be solved in the regions around Titan. Note in Eq .2 the electrical conductivity is assumed to be zero, a good assumption in the atmosphere of Titan. The eigen values of Eq. 3 must be found and might be measured by future space probes but due to the low conductivity of Titan's atmosphere lightning may rarely excite the Schumann resonances.

The Computer Programs
Variable permittivity

This is one of the many finite elements programs available but has the advantage of being free to use for the readers of this paper. It iscalled FlexPDE that can be obtained at www.PDEsolutions.com.

TITLE 'Natural Frequencies of Titan Ionosphere' {The relative permittivity, $\bullet$ is varied from 1 to 10}

COORDINATES
ycylinder VARIABLES H
SELECT

Quick Response Code

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cubic on errlim = 1e-05 modes = 4

DEFINITIONS c0 = 2.997925e08
JD = curl(H)*0.5/Pi ep0 = 8.854e-12 mu0 = Pi*4e-07 k =staged(1,1.5,2,2.53,3.5,4,5,6,7,8,9,10)
f = ((lambda/(mu0*ep0*k))^0.5)/(2*Pi) TR = 2575000 TA = TR + 720000 EQUATIONS
curl(curl(H)) = lambda*H BOUNDARIES

REGION 1
START(0,-TA) arc(center=0,0) angle = 180 line to (0,TR) arc(center=0,0) angle = -180 line to close

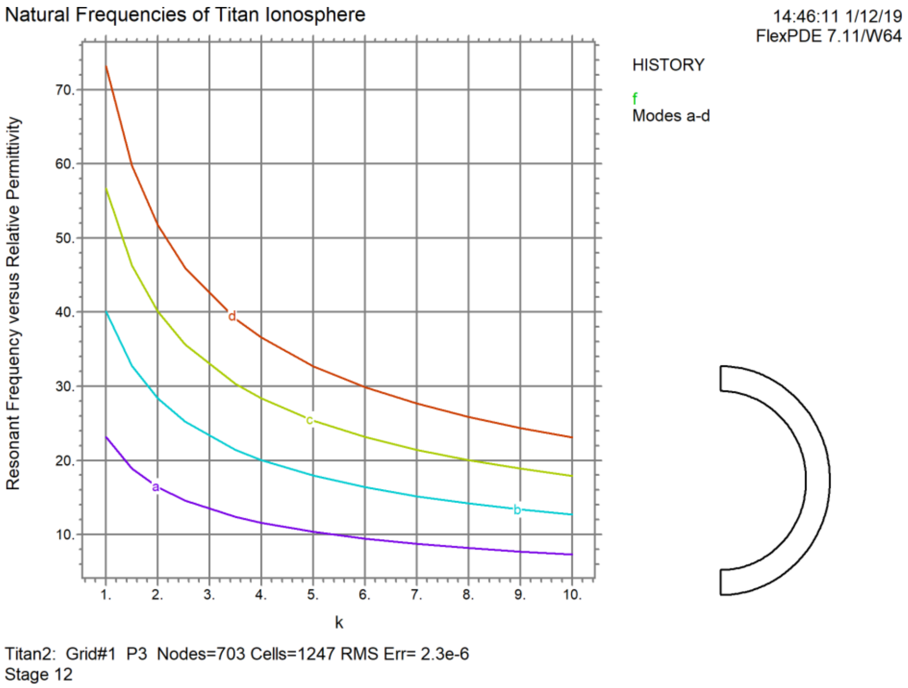

Figure 1: Schumann resonant frequencies vs □□

MONITORS
contour(H) painted as 'Azimuthal Magnetic Intensity' report f as 'resonant frequency' report k as 'Relative Permittivity'
PLOTS
CONTOUR(H) painted as 'Azimuthal Magnetic Intensity' report f as 'resonant frequency' report k as 'Relative Permittivity'
vector (JD) as 'Displacement Current' norm report f as 'resonant frequency' report k as 'Relative Permittivity'
HISTORIES
history(f) vs k as 'Resonant Frequency versus Relative Permittivity' END

In Fig1. is a plot of the Schumann resonant frequency as a function of relative permittivity, □□. The first two modes for □□ = 3 are about 14 and 23 Hertz assuming a spherical shape for Titan. Given the work of Marente et al these results are reasonable.

Variable ellipticity
{The oblateness of the Titan is varied from 0 to 1000000
km resulting in ellipticities as large as 0.25 and the relative permit- tivity is set to 3}

COORDINATES
ycylinder VARIABLES H
SELECT
cubic on

```
! errlim = 1e-05 modes = 4
    DEFINITIONS !Here the dimensions of the geometry and physical constants are defined.
c0 = 2.997925e08
JD = curl(H)*0.5/Pi k = 3 ep0 = 8.854e-12 mu0 = Pi*4e-07 f = ((lambda/(mu0*ep0*k))^0.5)/(2*Pi)
BR =staged(0,20000.0,40000.0,60000.0,70000.0,80000.0,90000.0,100000.0)
!staged Titan ellipticity
TR = 2575000 TA = TR + 720000 ep = (1-(TR/(TR+BR))^2)^0.5 EQUATIONS
    curl(curl(H)) = lambda*H !The electrical conductivity is assumed to be zero
BOUNDARIES REGION 1
    START(0,-TA) arc(center=0,0) to (TA+BR,0) !By adding BR to TA create an elliptical curve
arc(center=0,0) to (0,TA) line to (0,TR) arc(center=0,0) to (TR+BR,0) arc(center=0,0) to (0,-TR) line to close MONITORS
    contour(H) painted as 'Azimuthal Magnetic Intensity' report f as 'resonant frequency' report ep as 'Titan ellipticity'
PLOTS
    CONTOUR(H) painted as 'Azimuthal Magnetic Intensity' report f as 'resonant frequency' report ep as 'Titan ellipticity'
PLOTS
    vector (JD) as 'Displacement Current' norm report f as 'resonant  frequency' report ep as 'Titan ellipticity'
PLOTS HISTORIES
history(f) vs ep as 'Resonant Frequency versus Titan ellipticity'
```

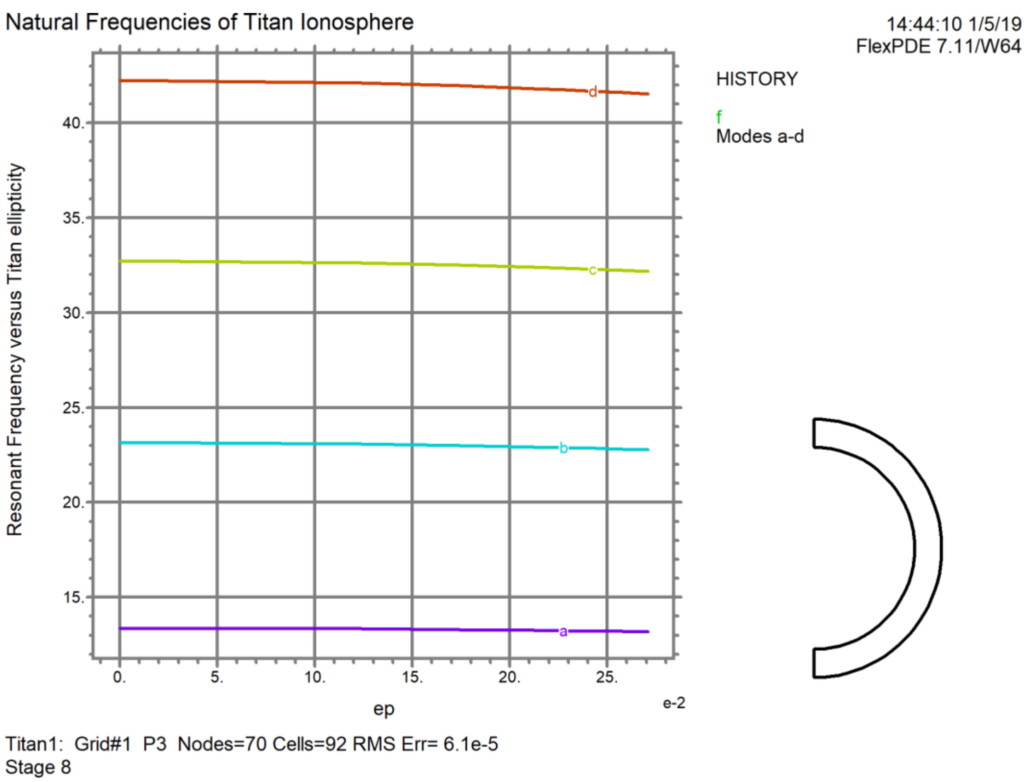

Figure 2: Schumann resonances vs ellipticity

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END
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The resonant frequencies do no vary much due to the ellipticity that may be present in Titan. The first resonant frequency is around 14.3 Hertz, the second 23 Hertz, the third 34.5 Hertz and the fourth 42 Hertz. These results for zero ellipticity compare well to the values given by Morente *et al.* (2003) that yields the lowest frequency in magnitude of 14.8, Clearly the resonant frequencies diminish as the ellipticity of Titan increases. The aforementioned programs produce too many graphs of magnetic intensity and displacement current distributions to exhibit here. For brevity one of each is shown. In Fig. 3 and Fig. 4 are shown the displacement current density and the azimuthal magnetic intensity when the ellipticity of Titan is 0.27. In this case the first resonant frequency is 13.19 Hertz.

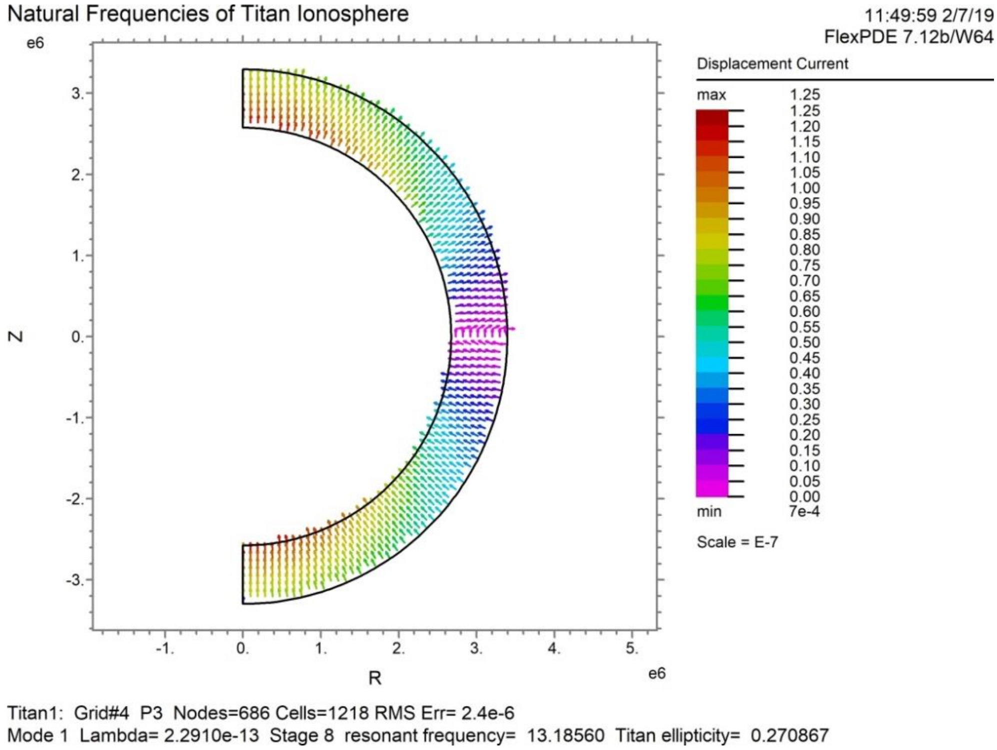

Figure 3: Displacement vectors

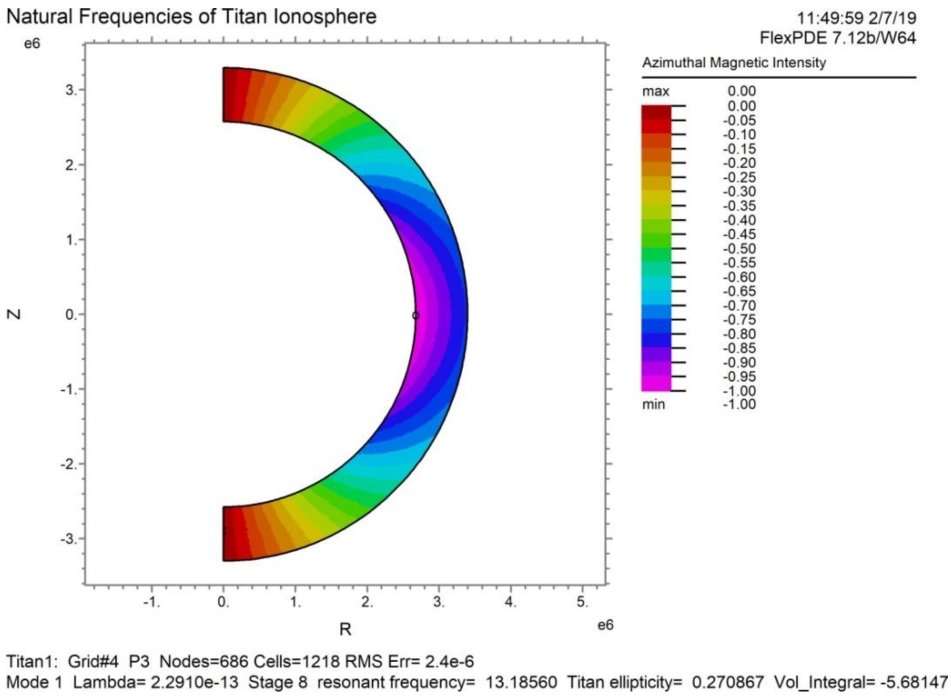

Figure 4: Azimuthal magnetic intensity

References

1. Morente, J.A., Molina-Cuberos, G.J., Portf, J.A., Schwingenschuh, K., Besser, B.P., (2003). A study of the propagation of electromagnetic waves in Titan's atmosphere with the TLM numerical method. *Icarus* 162, 374-384.