East African Scholars Journal of Engineering and Computer Sciences
Abbreviated Key Title: East African Scholars J Eng Comput Sci
ISSN 2617-4480 (Print) | ISSN 2663-0346 (Online) |
Published By East African Scholars Publisher, Kenya

OPEN ACCESS

Volume-2 | Issue-8 | Aug-2019 |

Research Article

Statistical Analysis of the Queuing System at the Opd in Bolgatanga Regional Hospital

Nyaaba Donatus[1*], Francis Atintono[1], Slyvester Alosum Anaba[1] and John Ayuekanbey Awaab[2]

[1]Bolgatanga Polytechnic, Department of statistics, P. O. Box 767, Ghana -West Africa
[2]University for Development Studies, Department of Statistics, P. O. Box 1350, Ghana,-West Africa

*Corresponding Author
Nyaaba Donatus

Abstract: This study reviewed the works done on queuing systems in several situations where customers had to wait in a queue to receive the services they needed. Statistical tools such as the repeated measure ANOVA, correlation analysis and chi square of Goodness of fit for Poisson distribution were employed to analyze the data collected across three designated points concerning the service times of patients, waiting times and inter- arrival time at the OPD of Bolgatanga Regional Hospital . These analysis were to assist to achieve the objectives of determining whether there is a significant linear relationship between the service times and waiting times of patients, establish whether the service times of patients across the three points in the department are the same and the waiting times as well, and finally, to ascertain whether the inter-arrival times follow a Poisson distribution. Results shown that the minimum and maximum service times at the first point were 2.22 minutes and 35.17 minutes respectively with a corresponding mean service time of 10.4545 minutes. The minimum and maximum waiting times at the vital taking point were 0.27 minutes and 16.082 minutes respectively with a mean waiting time of 3.3949 minutes. The correlation coefficient was -0.378 with a p-value of 0.000. For the Mauchly's test of sphericity for service times the Greenhouse Geisser and Huynh Feldt test statistics were 0.705 and 0.712 respectively whiles that of the waiting time were 0.544 and 0.546 respectively. The F-ratio and the Partial Eta squared values for the service times were 83.221 and 0.457 respectively ant that of the waiting times were 44.952 and 0.312. Finally, the results gave a chi square test statistic value of 451.132.

Keywords: repeated measure ANOVA, Mauchly's test of sphericity for service times the Greenhouse Geisser and Huynh Feldt test statistics.

BACKGROUND OF THE STUDY

The total population of the Bolgatanga metropolis was 223,252 in the 2010 population and housing census representing 9 percent of the total population of the Upper East Region of Ghana. This was made up of 11,109 males and 112,143 females constituting 49.8 and 50.2 percent respectively. The metropolis has a predominantly urban population constituting 80.2 percent. Generally, the metropolis depicts a youthful population. The population aged 0-14 years is 81,156 representing 36.4 percent whiles those within 15-64 years who constitute the total labor force are 131,826 representing 59 percent.

From the 2018 world population review, it is estimated that the population of the Bolgatanga municipality increased to 360,579 with a growth rate of about 3.5 percent. This rapid increase in population would have so many burdens on the social amenities in the area not excluding the health service delivery institutions. This is highlighted by the fact that the population of the area is dominated by women and children who are by far vulnerable to various chronic diseases. The sanitation situation in the metropolis is a cause for concern as the area produces a lot of waste which are not well managed. Majority of the household within the metropolis are without a toilet which force most of the residents to resort to open defecation which recently placed the Metropolis in the bottom of the open defecation league table. This practice pose serious health threat to the people.

Quick Response Code

Journal homepage:
http://www.easpublisher.com/easjecs/

Article History
Received: 13.07.2019
Accepted: 22.07.2019
Published: 09.08.2019

DOI: 10.36349/easjecs.2019.v02i08.002

Based on the above-mentioned factors, it stands to reason that the demand for various health services in the area will increase substantially. Therefore, the health service delivery institutions in and outside the area often record high attendance because of the proportion of people demanding for health service in those institutions.

Beside the various determinants of health service demand outlined above, naturally, everyone become ill at one point of time in his or her life but decision taken by people to seek for good health care is very critical since such decisions can positively or negatively affect health care delivery and by extension the socio-economic status of the country. The health of every nation according to Ardeson (1983) is everybody's concern but individually, everyone is responsible for his or her own health. Health is personal issue and therefore each person should take care of him or herself so that he or she can be healthy. This will satisfy the World Health Organization {WHO} definition of health that "Health is a state of complete physical, mental and social wellbeing and not merely an absence of disease and infirmity (Payne and Hahn 2002, p, 67).

When people are sick, health seeking decision according to Horver, Davis and Donatelle (1993) may be influenced by a variety of factors and may further be motivated by a complex interaction between values, beliefs and selected factors in the social environment. And also, the way and manner they are treated in the various health service delivery institutions is a serious factor that they consider in seeking for health care services. Considering the various factors including the implementation of the national health insurance policy in the country, health care demand and utilization in the country has been increased and Bolgatanga to be specific. Clinic, hospitals and other health centers as well as the various stakeholders in the health sector always try to put measures in place to ensure that the teeming number of people seeking for health services are treated diligently. Upon all the efforts made to minimize if not eliminate the challenges confronting the health delivery system in the country, some setbacks still persist especially with the issue of client satisfaction.

One of the many challenges facing the health sector in Ghana is the poor quality of health services that are offered clients in the various hospitals and health delivery institutions. Same is said when it comes to the Bolgatanga municipality. Although, the various health service providers always try to address some of the challenges but one challenge which seemed not to receive the attention of many clinics, hospitals and the various stakeholders is the long queues normally seen in the various health service delivery centers which cause delays in rendering the needed health services. People normally complain bitterly about the amount of time they normally had to wait before they are offered the services they needed.

As a result of this, clients seemed not to be satisfied with the way they are been handled whenever they are seriously in need of any kind of health service in the various hospitals because measures geared towards the reduction of waiting lines in the hospitals seemed not to be in place. Therefore, there is the need to ensure clients satisfaction by adopting measures aimed at reducing the amount of time clients normally spent in the various hospitals. This study will help bring to light the challenge of waiting in long queues in seeking health services in the various hospitals and propose measures to be adopted to reduce if not eliminate this growing canker. This will add to the gains made as far as enhancing quality and effective health service delivery is concern.

PROBLEM STATEMENT

The Bolgatanga Regional like any other health service delivery institution is a key stakeholder in strengthening the health delivery system of the country by making sure that timely health service needs of the clients are met. In view of this, the hospital wants to increase the number of attendances to generate enough money to help strengthen the financial status of the institution in order to be operational effective. Clients in their choice of which hospital to seek for health service is also dependent on how timely they will be attended to, hence will prefer the hospital which put measures in place to ensure their convenience.

And waiting in long queues in hospitals has now become a big challenge confronting the health delivery system in the country. This compel clients to remain reluctant to seek for health services which have a potential implication on the health care delivery system of the country as well as socio-economic status of the country since productivity greatly depends on the robustness or otherwise of the health delivery system of any country.

Therefore, to ensure an effective operation of the health service delivery institutions and by extension to ensure client satisfaction to achieve the goal of minimizing if not eliminate the challenge confronting the health delivery system of the country, it is worth managing our health service delivery institutions in a way to reduce waiting time of clients. In this work, the queuing system at the Bolgatanga Regional Hospital is analyzed using some suitable statistical tools.

1.3 RESEARCH QUESTIONS

This research will extensively provide answers to the following questions

- Is there any significant linear relationship between the service and the waiting times of patients at the various designated points in the OPD?

- Are the mean service times of patients at the OPD differ across the three designated points?
- Is there any significant difference between the mean waiting times of patients at the OPD across the three designated points?
- Do the inter-arrival times of patients follow a Poisson distribution?

OBJECTIVES OF THE STUDY

The objectives of the study include:

- To determine whether there is a significant linear relationship between the service and waiting times of patients.
- To determine whether the mean service times of patients differ across the three points at the OPD.
- To determine whether there is a significant difference in the mean waiting times of patients across the three point at the OPD.
- To determine whether the inter-arrival time gap at the OPD follows a Poisson distribution.

HYPOTHESES

This study will be based on the following hypotheses:

- **Null hypothesis**: The correlation coefficient is not significantly different from zero.
- **Alternative hypothesis**: The correlation coefficient is significantly different from zero.
- **Null hypothesis**: The mean service time of patients is the same across the three points.
- **Alternative hypothesis**: The mean service time is significantly different at one or more points
- **Null hypothesis**: The inter-arrival times at the department follow Poisson distribution.
- **Alternative hypothesis**: The inter-arrival times at the department do not follow a Poisson distribution

SIGNIFICANCES OF THE STUDY

- The findings of the study are intended to assist the administration of the Bolgatanga Regional Hospital to do a proper assessment of the performance of the department to. This will help improve client satisfaction.
- The findings of this study will help administration of the Bolgatanga Regional Hospital to gather more information about the queuing system at the department under consideration based on which an informed decision will be taken in relation to the number of servers needed to ensure client satisfaction.

LITERATURE REVIEW

INTRODUCTION

This section of the study is focused on the review of the relevant literature on the application of queuing theory in numerous areas. The application of queuing theory in the area of health care is extensively discussed. Research papers that highlights queuing systems, the rudimentary queuing theory and its characteristics and among others are also reviewed in this section.

ELATED WORKS ON APPLICATION OF QUEUING THEORY

Nafees (2007), analyzed queuing systems for an empirical data of supermarket checkout service units as an example. The model designed for this example is a multiple queues multiple server model. The study required an empirical data which involves arrival time in the queue of the checkout operating units, departure time and service time.

In Sharma *et al.*, (2013),queuing theory is the mathematical study of waiting lines and is very useful to define modern information technologies requiring innovations that are based on modelling analysis to deal as well as procedure of traffic control of daily life of human like telecommunications, reservation counters, supermarkets ,big bazaar, picture cinema, hall ticker t window and also to determine the systems of computer operators, computer performance, health service and airline ticket sales. The paper discussed the approach of queuing theory and theory models.

Duder and Rosenwein (2001), used queuing based ‘rule of thumb’ formulas to estimates the cost of abandonment and to determine the optimum number of operators. Whitt (2005), also used queuing analysis for staffing a call Centre, considering the proportion of servers present as a random variable. Cugnasca (2007), provide useful information to build availability model for computer used in airspace control Centre based on analytical models provided by queuing theories. The research queuing model to establish availability parameters related to a data Centre operations and its management issues.

May (2012), analyzed data from open pit gold mine and applied to a multichannel queuing model representative of the loading process of the haul cycle. She stated that one method of fleet selection involves the application of queuing theory to the haul cycle and most mining haul routes consist of four main component: loading, loading hauling, dumping and unloading hauling to return to the loader. The output of the model was compared against the actual truck data to evaluate the validity of the queuing model developed.

WOENSEL and VANDAELE (2007), presented the overview of different analytic queuing model for traffic on road network. They showed that queuing model can be used to adequately model uninterrupted traffic flows. An analytical application tool to facilitate the optimal positioning of the counting point on a highway was also provide in the paper. Brown (2012), aimed at increasing understanding of system variables on the accuracy of simple queuing model. A queuing model was proposed that combines G/G/1 modelling technique for rework with effective processing time techniques for machine availability and accuracy of this model was tested at varying level of rework, external arrival availability and machine availability. The research shows that the model performs best under exponential arrival pattern and can perform well even under high rework conditions. Generalization made with regards to the use of this tool for allocation of jobs to specific workers and/or machines based on known rework rate with the ultimate aim of queue time minimization.

Alfares (2009), presented the modelling and solution of real-life operator scheduling problems at a call Centre. Queuing and integer programming models were combined to minimize the total weekly labor cost while providing acceptable service level for each hour of each day for the week. The determine the optimum staffing levels and employee weekly work schedules for meeting a varying work load for each hour of the week. Queuing analysis was employed to data on the number and duration of calls in order to estimate minimum hourly labor demand.

Chiang et al (2013), presented two scheduling polies to stay away from the risk of over positioning and under positioning in both quality of server and service rate. They modelled a cloud server form as an M/M/R queuing model such that system congestion cost and balking event loss can be estimated analytically. A cost function is developed by taking system operating cost, working made cost, system congestion cost etc. into consideration. Simulation results shown that the optimum quality of service rate can be obtained to minimize cost.

WORKS RELATED TO QUEUING SYSTEMS

Gurumurthin and Benjaafar (2014), considered queuing system with multipole classes of customers and heterogeneous servers where customers have the flexibility of being processed by more than one server and servers possesses the capacity of processing more than one customer classification. They provided unified frame work for the modelling and analysis of these system under arbitrary. Customer and server flexibility and for rich sets of control policies that include customer/server-priority schemes for customer and server selection. Several insight into the effect of system configuration and control policies were generated. In particular, they examined the relationship between flexibility control policy and throughout under varying assumptions for system parameters.

Filipowiez and Kwiecien (2008), described queuing system and queuing networks which are successfully used for performance analysis of different systems such as computer communication, transportation, networks and manufacturing. Classical Markovian systems were incorporated with exponential service time and a poison arrival process and queuing systems with individual service. The researcher studied oscillatory queuing systems with Cox and Weibull service time distributed as an example of Markovian systems.

Schwaitz *et al.*, (2006), derived stationary distribution of joint queuing length and inventory process in explicit product form for various M/M/1 system with inventory under continuous review and different inventory management policies and with lost sales. They considered demand to be poison and service time to calculate the performance of measures of the respective systems.

Kwok (1999), investigated the waiting time or delay of a class of mobile communication system oriented queuing model. Both exact analytic and approximation algorithm techniques were used in order to obtain the exact and appropriate waiting distribution of the model. The researcher proposed different queuing model in order to describe different characteristics of mobile systems. Different server control mechanism was implemented in order to optimize channel utilization.

RELATED WORKS ON THE APPLICATION OF QUEUING THEORY IN HEALTH CARE SITUATIONS

A considerable results-oriented researches have been done and the outcomes of those researches has shown that queuing theory can be useful in real world health care situations and some reviews of these works are vividly dealt with in this section of the study.

McClain (1976),reviews research on models for evaluating the impact of bed assignment polies on utilization, waiting times and probability of turning away patients, Nosek and Wilson (2001),review the use of queuing theory In pharmacy applications with particular attention to improving customer satisfaction, Customer satisfaction is improved by predicting and reducing waiting times and adjusting staffing. Preater (2002), presents a brief history of the using of queuing theory in health care and points to an extensive bibliography of the research that list many papers (However, it provides a description of the application of results).

Green (2006a), presents the theory of queuing as applied in health care. She discusses the relationships among delays, utilization and the number of servers. The basic M/M/S model, its assumptions and extension and application of the theory to determine the required number of servers. Jacobson (2006), presents list of steps that must be done carefully to model each health care scenario successfully using simulation and warn about the slim margin of tolerable and the effect of such error in lost lives. Tucker et al (1999) and Kao and Tung (1981), use simulation to validate, refine or otherwise complement the results obtained by queuing theory.

Albin *et al.*, (1990), show how one can use queuing theory to get approximate results and use simulation to refine them. Tseythin (2009), modelled "Emergency Department (ED) to Internal Ward (IW) process" as a queuing system with heterogeneous servers' pools, where the pools represent the ward and servers the beds. This system was analyzed using various queuing architecture and routing policies in search for fairness and operational performance. This queuing system with a single centralized queue and several server pools, form an invented V-model.

Lakshmi and Lyer (2013), review the contribution and application of queuing theory in the field of health care management problems. They proposed a system classification of health care areas which are examined with the assistance of queuing models. The goal was to provide sufficient information to analysts who are interested in using queuing theory to model a health care process and who want to locate the details of the of the relevant model.

Obamin (2010), presented the results of a study that evaluate the effectiveness of the queuing model in identifying the ante-natal queuing system efficiency parameters. Tora optimization system was used to analyze the data collected from ante-natal care unit of a public teaching hospital in Nigeria over three weeks period. The study shown that pregnant mothers spent less time in the queue and system in the first week than during the other succeeding two weeks.

Fomundam and Hermann (2007) conducted a survey of queuing theory application in health care. This paper survey the contribution and application of queuing theory in the field of health care. This paper unlike other reviews mentioned above was to show the applicability of queuing theory from the perspective of health care organizations, thus, this survey summarized a range of queuing theory results in the following areas: waiting time and utilization analysis, system design and appointment systems. This covered processes that provide direct patient treatment and process that provide auxiliary services such as pharmacy and laboratory processes.

This survey also considered results for systems at different scales, including individual departments, health care facilities and regional health system. It also covers analytical queuing theory model applied directly to health care system. Finally, Silva *et al.*, (2014), described and examined 'chronos' used Queue Management System (QMS) at a pharmacy service Out Patient Consulting waiting Rooms (OCR) and to present the results after two years of use. They used a retrospective cohort study. Cohort A and B was made before and after QMS implementation. The main outcome measures used in the study were general data, activity record, patient consultation, average waiting time and appointment compliance. At QMS, patients arriving at the OCR, for which they have an appointment, confirm their arrival by placing their health card in a reader in OCR, which prints out a ticket with the room number and time of consultation, arrival time and correlative number. The pharmacist checks patients using the computer screen in the consulting room and clicks on call to notify the patients, who hear acoustic signal and see their number on a screen after attending to the patient

They concluded that, the QMS eliminated manual system for recording work done, provide information about openings and closing times. It also eliminates FIFO queues and provides real time information on the patients in the waiting room. Finally, it improves arrival flows, reduces unscheduled patients checking in and reduces waiting times.

PERFORMANCE MEASURES OF QUEUING SYSTEMS

According to William (2003), waiting lines consist of mathematical formulas and relations used to determine the operating characteristics of these lines. Among these features are:

- The probability that there is no item in the system.
- The average of the times in the waiting line.
- The average of the existent item in the system.
- The average time an item spends in the waiting lines.
- The average time an item spends in the system.
- The probability that an item has to wait for the system.

In Kamba et al (2012), approximation to performance measures in queuing system have received considerable attention because these measures have wide applicability. They proposed two methods to approximate the queuing characteristic of a GI/M/I system, the first method was parametric in nature. Using only the first three moments of the arrival distribution. Their second method treads the known path of approximating the arrival distribution, by a mixture of two exponential distribution and marching the first three moments. Numerical example and optimal analysis of performance measures of GI/M/I queues was provided to illustrate the efficiency of the methods and are compared with benchmark approximation.

Gkougkouli *et al.*, (2014), presented a new fuzzy approaches to queuing modelling to estimate the disadvantages of point estimation and relevant paradoxes, when calculating the efficiency of the queues, in case of unreliable queuing data available. Both poison rate and exponential service time were considered by using fuzzy estimator constructed from statistical data. The introduction of fuzzy estimators in performance measures of M/M/S queuing systems was presented to address central estimation issues under uncertainty.

RESEARCH METHODOLOGY

INTRODUCTION

This section gives a brief explanation on the research design, population, sample and sampling procedure, source of data, research instrument, data collected and data analysis. The probability distributions such as exponential and poison used in the analysis of queuing systems are extensively dealt with.

RESEARCH DESIGN

This study seeks to achieve the objective of determining the nature of mean service time, mean waiting time and the inter-arrival time of patients at the Out Patient Department of the Bolgatanga Regional Hospital . In order to achieve these objectives, it is required to have data concerning the arrival and service times of patients who seek for health care services at the department. As a result, the research was designed in a way to get that kind of data.

The data collection was conducted within five working days starting from 12th January to 16th January 2019. The department had three designated points which included the following.

- The first point was the registration center where patients are registered and given folder.
- The point where the vital statistics of patients are taken.
- The consultation where the patients meet the doctor on duty.

For this reason, the data collection for the research was only concentrated on patients who were passing through these three designated points. When it is clear that the patient is passing through the three designated points, he or she is followed and the arrival time, service time and departure time recorded. The patient arrival time was recorded immediately he or she entered and join the queue. A card designed by the researcher was given to the selected patient for the purpose of identification. When the patient is called to the point of registration, his or her service start time was recorded and the departure time at that point was also recorded before he or she moves to join a queue at the second point (point where vitals are taken) where the service start time and the departure time at that point were also recorded. The patient will finally be followed to the last point (Consultation). This entire process was repeated for all sampled patients.

STUDY AREA

This study was conducted at the Out Patient Department at the Bolgatanga Regional Hospital.

BRIEF HISTORY ABOUT BOLGATANGA REGIONAL HOSPITAL

This study was undertaken in the Out Patient Department (OPD) at the oldest government hospital in the Upper East Regionof Ghana. The Bolgatanga Regional Hospital was established in July 1929 to provide 24 hour health to the people in the metropolis and the entire Upper East Regionas a whole. The hospital provides services such as OPD, Pharmacy, Antenatal care, Laboratory services, Theater service, Ear, Nose and Throat services, Psychiatry and maintenance care and among others.

The hospital attends to over 100 000 out patients annually and admit 24 000 in patients and carry out over 150 000 laboratory diagnosis. It also attends to over 26 000 pregnant women yearly with close to over 5000 of them delivering in a year.

POPULATION

The population under consideration in this study was the patients arriving in the hospital for various kinds of health care services especially at the Out Patient Department (OPD). The interest was specifically on Out Patients who were supposed to pass through the three designated points in the department (Point of registration, point where vitals are taken and consultation).

SAMPLE AND SAMPLING PROCEDURE

SAMPLE

Sample is a part of a population selected for a research purpose. This selected sample is intended to represent the entire population of interest. In research it is mostly difficult to consider the entire population especially when dealing with a very population due to factors such as time constraints and inadequacy of resources hence it is always appropriate to take some part of the population that can truly represents the population under consideration.

This was the same situation as far as this research is concerned as the hospital under consideration provides diverse health care services to hundreds of out patients each day and therefore it was impossible to consider each and every patient who came to the hospital to receive health care service especially at the Out Patient Department (OPD).

In this study, a total of hundred out patients were selected which took place within a space of five working days from Monday to Friday where twenty (20) out patients were selected each day and their arrival times, service times and departure times were recorded in the process of receiving service.

SAMPLING PROCEDURE

In taking a sample, there are a lot of sampling procedure for use and depending on the nature of the study and other factors, the researcher chooses the appropriate sampling procedure to be employed in the study. In this study, a simple random sampling procedure was used in selecting the patients for the data collection.

WHY SIMPLE RANDOM SAMPLING PROCEDURE?

This sampling procedure was deemed appropriate for this study because of the following reasons:

- The data collection was based on patients who will go through all the designated points at the OPD (Point of registration, point where vital statistics were taken and Consultation), but not all the patients were to go through those points. As a result, the patient selection was based on whether he or she will go through all the three points determined from the conversation the researcher had with the patients.
- To save time.
- Inadequate resources.

SOURCE OF DATA

The data required in this research was the arrival times, the service times and the departure times of patients who were in need of health care services at the Out Patients Department (OPD) in the hospital. This data according to the hospital was not available and hence the researcher had to take the data on his own. Therefore, a primary data was used in this study.

RESEARCH INSTRUMENT

The research instrument used in this study are:

* Questionnaire
* Designed cards (with numbers)
* Stop watch

QUESTIONNAIRE

This was one of the key tools used in this research especially during the data collection process. A questionnaire was designed to facilitate the data collection. The questionnaire used was designed with seven columns. The first column was to take the arrival time of patients. The second and the third columns were to take the service start times and the departure times of patients at the first point respectively. The fourth and the fifth columns were to take the service start times and the departure times of patients at the second point respectively. The last two columns were to contain the service start times and the departure times of patients recorded at the last point (Consultation).

DESIGNED CARDS

Rectangular cards were designed by the researcher to be used for the purpose of identification. The patients who were selected were given these cards to differentiate them from the patients who were not selected. This tool helped to identify the selected patients so as to reduce lost to follow up in recording the arrival, service and departure times of patients.

STOP WATCH

This was another important tool used during the data collection process which facilitate the time recording.

DATA COLLECTED AND DATA ANALYSIS

The data collected in this research was about the arrival times, service times and the departure times of patients who came to the hospital to receive various kinds of health care services especially at the OPD. The data analysis was carried out using SPSS and MILITAB by employing statistical tools such as repeated measure ANOVA, correlation analysis and the chi square Goodness of fit test. The probability distributions commonly used in analyzing queuing systems such as poison and exponential distributions were applied.

REPEATED MEASURE ANOVA

Repeated measure ANOVA is also called the between subject's ANOVA which is equivalent to one-way ANOVA applicable to related groups. This statistical tool is used test in order to detect any overall difference between related means. This particular test one independent variable and one dependent variable. The dependent variable needs to be continuous (interval or ratio) and the independent variable categorical (either nominal or ordinal). The repeated measure ANOVA can be used for two types of study designs. The studies that investigate either change in mean scores over three or more time points or difference in mean scores under three or more different conditions. In a repeated measure ANOVA, the independent variable has categories called levels or related groups where measurements are repeated over time.

Repeated measure ANOVA test the null hypothesis that the population means are equal against the alternative hypothesis that the population means differ significantly.

CORRELATION ANALYSIS

Correlation is a method of statistical evaluation used to study the strength of a relation between two numerically measure continuous variables. This particular analysis is useful when the researcher wants to establish if there are possible connection between variables. If correlation is found between variables it means that when there is a systematic change in one variable, there is also a systematic change in the other variable. If there is correlation found, depending on the numerical values measured, this can either be positive or negative. Positive correlation exists if one variable increases simultaneously with the other while negative correlation exists if one variable decrease when the other increases.

Pearson correlation coefficient is the measure of correlation and ranges between +1 and -1, therefore the closer the coefficient to either of these numbers the stronger the correlation of the data it represents. This statistical analysis tests the null hypothesis that there is no significant relationship against the alternative hypothesis that there is a significant relationship.

CHI SQUARE OF GOODNESS OF FIT

Chi square Goodness of fit test is a non - parametric test that is use to determine how the observed value of a given phenomenon is significantly different from the expected value. In chi square Goodness of fittest, the term Goodness of fit is used to compare the observed sample distribution with the expected probability distribution. Chi square Goodness of fit test determine how theoretical distribution (such as normal, binomial or Poisson) fits the empirical distribution. Chi square Goodness of fit test the null hypothesis that there is no significant difference between the observed and the expected value against the alternative hypothesis that there is a significant difference between the observed and the expected values.

POISON DISTRIBUTION

This is the distribution considered among the discrete probability distribution for count of events that occur randomly in a given interval. It is the most applicable in queuing theory. Generally, if X is the number of events in a given interval and λ is the mean number of events per interval, then the probability of observing x events in a given interval is given as:

$$P(X = x) = \frac{\lambda^x exp^{-\lambda}}{x!}$$

In queuing theory, the arrival rate of customers to a service facility is said to follow a poison distribution.

EXPONENTIAL DISTRIBUTION

The simplest queuing model assumed that the service time follows an exponential distribution. That is if λ is the mean arrival rate, then the probability density function for the time between successive intervals would be:

$$F(x) = \lambda e^{-\lambda t}$$

RESULTS AND ANALYSIS
DESCRIPTIVE STATISTICS
SERVICE TIMES

Table 4.1 shows the descriptive statistics of the service times of patients recorded at the three designated points at the OPD. The first column shows the total number of patients selected for the study. The second and the third columns take the minimum and the maximum service times respectively. The fourth and the third columns also contain the mean and standard deviation of the service times of patients respectively.

Table 1 Summary of the descriptive statistics of the service times of patients

	N	Minimum	Maximum	Mean	Standard deviation	Skewness	
	Statistic	**Statistic**	**Statistic**	**Statistic**	**Statistic**	**Statistic**	**Std.Error**
Service times at the registration point	100	2.22	35.17	10.4545	5.80668	2.562	.241
Service times at the vital taking point	100	.83	11.78	3.2747	1.31954	2.629	.241
Service times at the consultation point	100	1.25	21.18	9.1964	3.78272	.919	.241
Valid N (listwise)	100						

From the results in Table 4.1, the minimum service time at the registration point was 2.22 minutes and that of the maximum service time was 35.17 minutes. The mean and the standard deviation of the service times at the registration point were approximately 10.10.45 and 5.81 respectively. The registration point had the largest mean service time whiles the vital taking point had the least mean service time.

WAITING TIMES

Table 4.2 shows the descriptive statistics of the waiting times of patients recorded at the three designated points at the OPD. The first column shows the total number of patients selected for the study. The second and the third columns take the minimum and the maximum waiting times respectively. The fourth and the third columns also contain the mean and standard deviation of the waiting times of patients respectively.

Table 2 Summary of the descriptive statistics of the waiting times of patients

	N	Minimum	Maximum	Mean	Std.Deviation	Skewness	
	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Std.Error
Waiting times at the registration point	100	2.03	66.08	17.0479	8.97907	1.839	.241
Waiting times at the vital taking point	100	.27	16.02	3.3949	2.59741		.241
Waiting times at the consultation	100	1.57	141.57	28.9595	30.67158	1.929	.241
Valid N (listwise)	100						

From the results in Table 4.2, the minimum waiting time at the registration point was 2.03 minutes and that of the maximum service time was 66.08 minutes. The mean and the standard deviation of the waiting times at the registration point were approximately 17.05 and 8.98 respectively. The consultation point had the largest mean waiting time whiles the vital taking point had the least mean waiting time.

Correlation Analysis of the Service Times and the Waiting Times Of Patients

This analysis seeks to establish whether there is a significant correlation between the service times and waiting times of patients. This is done by testing the null hypothesis that there is no significant relationship between the service times and waiting times of patients against the alternative that there is significant relationship between the service times and waiting times of patients. Table 4.3 shows the Pearson coefficient and the p-value.

Table. 3 Summary of the correlation analysis of service and waiting times of patients

		service time of patients	waiting time of patient
service time of patients	Pearson Correlation	1	.378**
	Sig. (2-tailed)		.000
	N	300	300
waiting time of patient	Pearson Correlation	.378**	1
	Sig. (2-tailed)	.000	
	N	300	300

From the results in Table 4.3, the Pearson correlation coefficient is 0.378 which depicts a relatively weak positive correlation with a p-value of 0.000 indicating statistically significant relationship since the value is below 0.05. Therefore, the null hypothesis is rejected and conclude that there is significant relationship between the service times and the waiting times of patients.

REPEATED MEASURE ANOVA TES\T

Service Times

Sphericity Test

This analysis is to test whether the mean service times of patients across the three designated points within the Out Patient Department are the same. This is done by testing the null hypothesis that the mean service times of patients across the three points are the same against the alternative hypothesis that the mean service times of the patients across the three points significantly differ. The table 4.4 the Mauchly's test of sphericity.

Table 4: The output of Mauchly's test of sphericity of the service times of patients across the three points.

Within Subject Effect	Mauchly's W	Approx. Chi-Square	Df	Sig.	Epsilon[a]		
					Greenhouse Geisser	Huynh-Feldt	Lowerbound
factor1	.582	53.093	2	.000	.705	.712	.500

This Table test whether or not the assumption of sphericity have been met. This assumption is about the equality of the variance of the difference between all the three points within the department. From the table the approximated chi square value is 53.093 with a p-value of 0.000 which indicates a highly statistically significance and therefore the assumption is violated which needs to be corrected. The last columns from the table give the various test statistics for the correction which include the Greenhouse-Geisser and Huynh-Feldt. The results have shown a test statistics values of less than 0.75; as a result Greenhouse-Geisser will be used for the correction.

Table 5 Output of test of within subjects' effect for the service times of patients

Source		Type III Sum of Squares	Df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
factor1	Sphericity Assumed	2906.434	2	1453.217	83.221	.000	.457	166.443	1.000
	Greenhouse-Geisser	2906.434	1.410	2061.064	83.221	.000	.457	117.356	1.000
	Huynh-Feldt	2906.434	1.424	2040.326	83.221	.000	.457	118.549	1.000
	Lower-bound	2906.434	1.000	2906.434	83.221	.000	.457	83.221	1.000
Error(factor1)	Sphericity Assumed	3457.486	198	17.462					
	Greenhouse-Geisser	3457.486	139.606	24.766					
	Huynh-Feldt	3457.486	141.025	24.517					
	Lower-bound	3457.486	99.000	34.924					

Test of Within Subjects Effects for Service Time

From the results in Table 4.5, the degree of freedom is given as 1.410 and 139.606 which represent the size of the sample. The F-ratio of 83.221 is large enough to draw conclusion that the independent variable (the designated point) is more likely to have a significant influence on the dependent variable (Service time). This significant effect implies that, there is less unexplained variance in the data than can be accounted for by the independent variable (the designated points). The p-value of 0.000 indicates a highly significance since is less than 0.05 which further confirms the significance of the F-ratio. This smaller p-value means that the null hypothesis is rejected, inferring that the results did not occurred by chance or mas the results of sampling error but due to the influence or effect of the independent variable (the designated points).

Finally, the Partial Eta squared which gives an idea of the magnitude of the effect or influence of independent variable (designated point) by measuring the effect size produce a value of 0.457. This value measure how much an effect or influence the designated points in the OPD has on the service times of patients. Since the value is more than 0.14, then it can be concluded that there the effect of influence of the designated points on the service times of patients is large enough.

Pairwise Comparison of the Mean Service Times

Table 5 shows the pairwise comparison of the mean service times at the designated points at the Out Patient Department. It shows the mean difference. Standard error and the p-value between the points being compared.

Table 6. The output of the pairwise comparison of the mean service times at the designated points.

(I) factor1	(J) factor1	Mean Difference (IJ)	Std. Error	Sig.[a]	95% Confidence Interval for Difference[a]	
					Lower Bound	Upper Bound
1	2	7.141*	.559	.000	6.033	8.250
	3	1.258	.749	.096	-.228	2.744
2	1	-7.141*	.559	.000	-8.250	-6.033
	3	-5.883*	.418	.000	-6.713	-5.053
3	1	-1.258	.749	.096	-2.744	.228
	2	5.883*	.418	.000	5.053	6.713

The results from Table 4.6 indicate the mean difference of 7.141 and 1.258 when the registration point is compared with the vital taking points and the consultation respectively. It also shows the mean difference when the vital taking point is compared with consultation with a value of -5.883. The p-values clearly indicates that all the possible pairs are statistically significant except for registration compared to consultation whose p-value is 0.096 which is more than 0.05 and hence insignificant.

Mean Plot of the Service Time.

The diagram shows a mean plot of the service time of patients at the various designated points. The estimated marginal means are plotted against designated points in the department. The horizontal axis takes designated point and that of the vertical axis takes estimated marginal means.

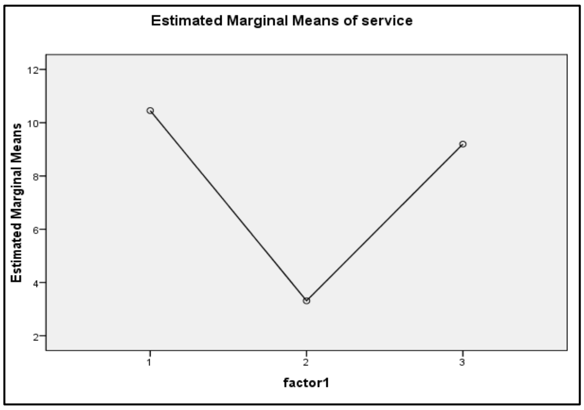

Figure 4.1 shows the output of the mean plot for the service times

The results from Figure 4.1 indicates that patients spend more time in receiving service at the registration point followed by the consultation and spend least time at the vital taking point.

Waiting Times

Sphericity Test

This analysis is to test whether the mean waiting times of patients across the three designated points within the Out Patient Department are the same. This is done by testing the null hypothesis that the mean waiting times of patients across the three points are the same against the alternative hypothesis that the mean waiting times of the patients across the three points shows significantly difference. The table 4.7 the Mauchly's test of sphericity.

Table. 7 The output of Mauchly's test of sphericity of the waiting times of patients across the three points

Within Subjects Effect	Mauchly's W	Approx. Chi-Square	Df	Sig.	Epsilon[a]		
					Greenhouse-Geisser	Huynh-Feldt	Lower-bound
Points	.162	178.093	2	.000	.544	.546	.500

This Table test whether or not the assumption of sphericity have been met. This assumption is about the equality of the variance of the difference between all the three points within the department. From the table the approximated chi square value is 178.093 with a p-value of 0.000 which indicates a highly statistically significance and therefore the assumption is violated which needs to be corrected. The last columns from the table give the various test statistics for the correction which include the Greenhouse-Geisser and Huynh-Feldt. The results has shown a test statistics values of 0.544 and 0.546 for Greenhouse Geisser and Huynh Feldt respectively which are less than 0.75, as a results Greenhouse-Geisser will be used for the correction.

TEST OF WITHIN SUBJECTS EFFECTS FOR WAITING TIME

Table 8 Output of test of within subject's effect for the waiting times of patients

Source		Type III Sum of Squares	Df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
Points	Sphericity Assumed	32727.882	2	16363.941	44.952	.000	.312	89.903	1.000
	Greenhouse-Geisser	32727.882	1.088	30069.281	44.952	.000	.312	48.926	1.000
	Huynh-Feldt	32727.882	1.091	29992.430	44.952	.000	.312	49.051	1.000
	Lower-bound	32727.882	1.000	32727.882	44.952	.000	.312	44.952	1.000
Error(points)	Sphericity Assumed	72078.721	198	364.034					
	Greenhouse-Geisser	72078.721	107.753	668.924					
	Huynh-Feldt	72078.721	108.029	667.215					
	Lower-bound	72078.721	99.000	728.068					

From the results in Table 4.8, the degree of freedoms is given as 1.088 and 107.753. The F-ratio of 44.952 is large enough to draw conclusion that the independent variable (the designated point) is more likely to have a significant influence on the dependent variable (Waiting time). This significant effect implies that, majority of variance in the data are accounted for by the independent variable (the designated points). The p-value of 0.000 indicates a highly significance since is less than 0.05 which further confirms the significance of the F-ratio. This smaller p-value means that the null hypothesis is rejected, meaning the results did not occurred by chance or as the results of sampling error but due to the influence or effect of the independent variable (the designated points).

Finally, the Partial Eta squared produce a value of 0.312. This value measure how much an effect or influence the designated points in the OPD has on the waiting times of patients. Since the value is more than 0.14, then it can be concluded that the effect or influence of the designated points on the waiting of patients is large enough.

Pairwise Comparison of the Mean Waiting Times

Table 10: shows the pairwise comparison of the mean waiting times at the designated points in the OPD. It shows the mean difference, standard error and the p-values between points being compared.

(I) points	(J) points	Mean Difference (I-J)	Std. Error	Sig.[a]	95% Confidence Interval for Difference	
					Lower Bound	Upper Bound
1	2	13.653*	.824	.000	12.017	15.289
	3	-11.912*	3.371	.001	-18.601	-5.222
2	1	-13.653*	.824	.000	-15.289	-12.017
	3	-25.565*	3.130	.000	-31.775	-19.354
3	1	11.912*	3.371	.001	5.222	18.601
	2	25.565*	3.130	.000	19.354	31.775

The results from Table 4.9 indicate the mean difference of 13.653 and -11,912 when the registration point is compared with the vital taking points and the consultation respectively. It also shows the mean difference when the vital taking point is compared with consultation with a value of -25.565. The p-values clearly indicates that all the possible pairs are statistically significant since there are all less than 0.05.

Mean Plot of the Waiting Time.

The diagram shows a mean plot of the waiting time of patients at the various designated points. The estimated marginal means are plotted against designated points in the department. The horizontal axis takes the designated point and that of the vertical axis takes the estimated marginal means.

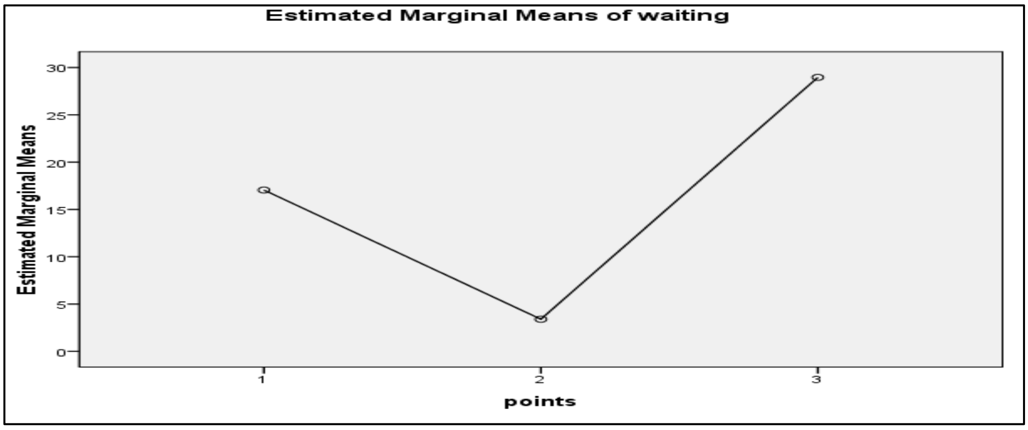

Figure 4.2 shows the output of the mean plot for the waiting times

The results from Figure 4.2 indicates that patients spend more time waiting for service at the consultation followed by the registration and spend least time at the vital taking point.

Chi Square Test of Time Gap between Arrivals for Poisson distribution

This analysis is to determine whether or not the inter-arrival times of the patients follow a poisons distribution. This is done by testing the null hypothesis that the inter-arrival times follow a Poisson distribution against the alternative that the inter-arrival times do not follow a Poisson distribution.

Table 11: Output of Goodness-of-Fit Test for Poisson distribution

Inter-arrival times	Observed	Probability	Expected	Contribution to chi square
<=17	57	0.476152	47.6152	1.850
18	0	0.093581	0.3581	9.358
19-20	1	0.167343	16.7343	14.794
21-22	2	0.122452	12,2452	8.572
23-24	2	0.074805	7.4805	4.015
25-26	2	0.038720	3.8720	0.905
>-27	36	0.026947	2.6947	411.638

Table 12: shows the test statistic of the chi square of Goodness of fit, the degree of freedom and the p-value.

Table 12: shows the chi square test statistic

N	N*	Df	Chi square	p-value
100	0	5	451.132	0.000

The results show a chi square test statistic of 451.132 with a p-value of 0.000 which indicates statistically significance and therefore the null hypothesis is rejected and conclude that the inter-arrival times of patients do not follow a Poisson distribution hence does not Goodness of fit.

DISCUSSION OF RESULTS

Table 1 displays the descriptive statistics of the service times of patients at the OPD. From the results, the minimum service time at the registration point was 2.22 minutes whiles the maximum service time was 35.17 minute. For the vital taking point, the minimum and the maximum service times were 0.83 minutes and 11.78 minutes respectively and also the consultation recorded 1.25 minutes and 21.18 minutes as the minimum and maximum service times respectively. This implies that the highest service time was recorded at the registration point whiles the lowest was recorded at the vital taking point. The mean service time recorded at the three points includes 10.4545 minutes, 3.2747 minutes and 9.1964 minutes respectively which indicates that patients spend on average 10.4545 minutes in receiving service at the first point which is the highest and spend on average 3.2747 minutes in receiving service at the second point which represents the lowest.

Table 2 displays the descriptive statistics of the waiting times of patient s at the OPD. The results indicate that the minimum and the maximum time a patient spends waiting for his turn to be served at the registration point are 2.03 minutes and 66.08 minutes respectively. The vital taking point also record a minimum waiting time of 0.27 minutes and a maximum of 16.082 and the consultation also recorded a minimum and maximum waiting times of 1.57 minutes and 141.57 minutes respectively. This means that the highest number of minutes a patient spends waiting for service was recorded at the consultation whiles the minimum was recorded at the vital taking points. The mean waiting time of patients at the various points in the department were 17.079 minutes, 3.3949 minutes and 28.9595 minutes respectively. This indicates that a patient spends on average 28.9595 waiting for his turn to be served at the consultation which was the highest followed by the registration point and the vital taking point.

Table 3 displays the output of the correlation analysis between the service times and the waiting times of patients. The results indicate a correlation coefficient of 0.378. This implies that there is a positive linear relationship between the service and waiting times of patients as one increases the other also seem to increases and the vice versa. The results also indicate a p-value of 0.000 which is highly significant and therefore the null hypothesis is rejected and conclude that there is a significant relationship between the service times and waiting times of patients.

Table 4 displays an output of Mauchly test of sphericity which indicates the violation of the assumption of equality of variance of the difference between all the designated points and hence correction is required using the Greenhouse Geisser statistic for the interpretation of the results of within subject effect output. This is because, both the Greenhouse Geisser and Huynh Feldt statistics are less than 0.75.

Table 5 displays the within subject effect output. This result shows a degree of freedom of 1.410 and 139.606 under the factor and error (factor) row respectively corresponding to Greenhouse Geisser. The F-ratio is given as 83.221 which is large enough to conclude that independent variable (the designated point) is more likely to have a significant influence or effect on the dependent variable (Service time). This significant influence or effect implies that majority of the variation in the mean service times across the points are accounted for by the independent variable (the designated point).The p-value of 0.000 is less than 0.05, therefore the null hypothesis that the mean service times across all the three designated points in the department are the same is rejected and conclude the mean services times across all the points differ significantly.

Table 6 displays a pairwise comparison of the three points to establish where the difference in the mean service time can be located. These results showed the mean difference of 7.141 and 1.258 when the first point is compared with the second and the third points respectively and also a mean difference of 5.883 when the second point is compared with the third point. Apart from the registration point compared with the consultation which produce a p-value of 0.096 which appeared to insignificant. The rest of the possible pairs produce a p-value less than 0.05 hence indicates statistical significance. Therefore, it can be concluded that the mean service difference come from all the possible pairs except for registration versus consultation.

Figure 1 shows the plot of estimated marginal means of service times of patients. The plot indicates that patients spend more time in the process of receiving the needed service at the registration point whiles spend the least average time at the vital taking points.

Moreover, Table 7 displays an output of Mauchly test of sphericity which indicates the violation of the assumption of equality of variance of the difference between all the designated points and hence correction is required using the Greenhouse Geisser statistic for the interpretation of the results of within subject effect output. This is because, both the Greenhouse Geisser and Huynh Feldt had a statistic value of 0.544 and 0.546 respectively which are both less than 0.75.

Table 8 displays the within subject effect output, producing a degree of freedoms of 1.088 and 107.753 under the factor and error (factor) row respectively corresponding to Greenhouse Geisser. The F-ratio is gave a value of 44.952 which is large enough to conclude that the independent variable (the designated point) is more likely to have a significant influence or effect on the dependent variable (Waiting time). This significant influence or effect implies that majority of the variation in the mean waiting times across the points are accounted for by the independent variable (the designated point). The results also gave Partial Eta squared value of 0.312 which indicates the size of the effect or influence of the designated point on the waiting times of patients. The p-value of 0.000 is less than 0.05, therefore the null hypothesis was rejected and concluded that the mean waiting times across all the points differ significantly.

Table 9 displays a pairwise comparison of the three points to establish where the difference in the mean waiting time can be located. This result shows the mean difference of 13.653 and 11.912 when the first point is compared with the second and the third points respectively and also a mean difference of 25.565 when the second point is compared with the third point. All the possible pairs produce a p-value less than 0.05 hence indicates statistical significance. Therefore, it can be concluded that the mean waiting time difference came from all the possible pairs.

Figure 2 shows the plot of estimated marginal means of waiting times of patients. The plot indicates that patients spend more time in waiting for their turn to be served at the consultation whiles spend the least average time at the vital taking points.

Finally, Table 10 indicates a chi square Goodness of fit for Poisson distribution to ascertain whether the inter-arrival time gap between patients follows a Poisson distribution. The results produce a chi square test statistic of 451.132 and a p-value of 0.000 which shows a statistically significance, therefore we reject the null hypothesis and conclude that the inter-arrival times do not follow a Poisson distribution.

CONCLUSION

This studied the queuing system at the Out Patient Department is the Bolgatanga municipality to help achieve the objectives of determining the significant linear relationship between the service times and waiting times of patients, determine whether the mean service times of patients across the three designated points at the department are the same and as well as the waiting times of patients and finally to determine whether the inter-arrival times of patients follows a poison distribution.

A primary data concerning the service times, waiting times and inter-arrival times of selected patients were recorded within five working days at the OPD. This data was analyzed using SPSS and MINITAB employing three statistical tools such as repeated measure ANOVA, correlation analysis and chi square Goodness of fit test for Poisson distribution.

After a successful conduct of this study, the following are the research findings of the study:

- There was a significant linear relationship between service times and waiting times of patients
- There was a significant difference in the mean service times of patients across the three designated points at the department. This may be as a results of the difference in services offered and also due to the use of the HAMS software which sometimes develop some challenges or the user is not all that conversant with the usage.
- The mean waiting times of patients across the three points in the department differ significantly.
- The inter-arrival times at the department appeared not to follow a Poisson distribution this may be as a result of irregular arrivals of patients to department due to a perceive long waiting times.
- Patients spend more time in the process of receiving service at the registration point and that of the consultation.
- Patients spend more time waiting for their turn to be served at the consultation and that of the registration point.

RECOMMENDATION

Based on the findings of the study the following recommendation is proposed:

- Monthly workshop should be organized for the worker in the department to keep them updated on how to use the Hospital Administration Management System (HAMS) software at both the registration point and the consultation.
- Open forum or a medium should be created for patients to assessment the performance of the department since it receives majority of the number of patients who visit the facility.
- More seats should be provided for the patients waiting for their turns to be served.
- Enough funds should be allocated to the fueling of the power plant in the facility.

REFERENCES

1. Acheampong, L. (2013). Queuing in health care centres, a case studu of outpatient department of south suntreso hospital. Master’s thesis, Kwame Nkrumah University of Science and Technology.
2. Alfares, H. K. (2009). Operator scheduling using queuing theory and mathematical programming models. Technical report, King Fahd University of Petroleum and Minerals.
3. Brown, A. J. (2012). A study of queuing theory in low to high rework environments with process availability. Master’s thesis, University of Kentucky.
4. Chiang, Y.-J., Ougang, Y.-C., & Hsu, C.-H. (2013). An optimal cost-ecient resource provisioning for multi-servers cloud computing. Cloud Computing and Big Data, 3:225231.
5. Cugnasca, P. S. (2007). Application of queuing theory for availability assessment in airspace control systems. JOURNAL OF THE BRAZILIAN AIR TRANSPORTATION RESEARCH SOCIETY, 3:158.
6. Duder, J. C., & Rosenwein, M. B. (2001). Toward zero abondements in call center performance. European Journal of Operations Research, 1:5056.
7. Filipowicz, B., & Kwiecien, J. (2008). Queuing systems and networks, models and applications. Bulletin of Polsh Academy of Sciences, 56.
8. Gkougkouli, A., Sris, D. S., Botzoris, G., & Papadopoulos, B. (2014). Fuzzy performance measures of m/m/s queuing systems using fuzzy estimators. Econpapers, XVIII:317.
9. Kamba, N. S., Rangan, A., & Moghimihadji, E. (2012). Approximations of performance measures in queuing systems. South African Journal of Industrial Engineering (online), 23:3041.

10. Kwok, W. W. P. (1999). A variety of queuing models for mobile communication systems. Master's thesis, University of Toronto.
11. Lakshmi, C., & Lyer, S. A. (2013). Application of queuing theory in health care.Operational Research for Health, 2:2539.
12. May, M. A. (2012). Application of queuing theory for open -pit truck/shovel haulage systems. Master's thesis, Virginia Polytechnic and State University.
13. Nafees, A. (2007). Analysis of the sales checkout operation in ica supermarket using queuing simulation. Master's thesis, University of Dalarna.
14. Obamiro, J. K. (2010). Queuing theory and patient satisfaction. Bulletin of Petroleum-Gas University of Ploiesti, LXII: 111.
15. Schwartz, M., Sauer, C., and Daduna, H. (2006). Queuing systems with inventory. Queuing Systems, 54:5578.
16. Sharma, A. K., Kumar, R., & Sharma, G. K. (2013). Queueing theory approach with queueing model. International Journal of Engineering Science Invention, 2:0111.
17. Silva, P., Framinan, L. M., Hesmida, J. V., & Molares, J. (2014). A computerized queue management system in the outpatient pharmaceutical care unit of a hospital phamacy service. European Journal of Hospital pharmacy, 21(1).
18. Tseytlin, Y. (2009). Queung systems with heterogeneous servers: on fair routing of patients in emergency departments. Master's thesis, Isreal Institute of Technology.
19. William, A. S. (2003). An Introduction to Management Science-Quantitative Approach to Decision Making. South-Western College Pub; 12 edition.
20. WOENSEL, T. V., & VANDAELE, N. (2007). Modelling trac ows with queuing models. Asia-Pacic Journal of Public Health Journal of Operational Resear ch, 24:435.