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Mathematics

Original Research Article

A Probabilistic Analysis of India's Patriotic Song *Aye Mere Watan Ke Logo*

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Abstract: This paper deals with a probabilistic analysis of one of India's most popular and historically important patriotic songs of all time, namely, *Aye Mere Watan Ke Logo*. The analysis of the song concerns with verifying whether the notes in the song are having fixed probabilities throughout the song or are the note probabilities varying. This is important as it is a determining factor to whether the probability distribution followed by the notes is multinomial or quasi multinomial respectively. A Chi Square goodness of fit is performed which confirms the multinomial model.

Keywords: Multinomial distribution; Chi-square goodness of fit test; probabilistic analysis

1. Introduction

Patriotic song: *Aye Mere WatanKe Logo* ("O people of my country") is a Hindi patriotic song written by the lyricist Pradeep, the tune being composed by C. Ramchandra and was sung by the legendary playback singer Lata Mangeshkar. The song commemorates the Indian soldiers who died during the Sino-Indian War in 1962. It is a tribute to the martyrs; the song enabled the young and the old to control and overcome the anger of defeat. This song is often presented at patriotic occasions in India [3]. The analysis of the song concerns with verifying whether the notes in the song are having fixed probabilities throughout the song or are the note probabilities varying. This is important as it is a determining factor to whether the probability distribution followed by the notes is multinomial or quasi multinomial respectively. A Chi Square goodness of fit is performed which confirms the multinomial model.

2. Research Methodology

2.1 Multinomial Distribution [4]

It is a generalization if the binomial distribution. The multinomial distribution arises from the extension of Binomial distribution where for *n* independent trail has *k*>=2 possible outcomes (Binomial distribution is its particular case for k = 2). In a multinomial distribution, on any given trial, the probability that a particular outcome will occur is constant.
Probability mass function of Multinomial Distribution:

$$\frac{n!}{x_1 ! \dots \dots \dots x_k!} p_1^{x1} \dots \dots p_k^{xk}$$

This gives the probability that out of n independent trials, event E_i (with fixed probability p_i) will occus x_i times, i = 1, 2….k. Obviously we must have $\sum x_i = n$, $0<p_i<1$, i = 1, 2, …k

2.2 Chi Square Goodness of fit test [1]

The square of a standard normal variate is a Chi-Square variate with one degree of freedom. Chi Square distribution exhibits additivity property, i.e., the sum of independent Chi-Square variates is also a Chi-Square variate with degrees of freedom added.

In our case Chi-Square = $\sum$(1/Expected frequency) * (Observed frequency – Expected frequency)^2 which follows ChiSquare distribution with k-1 degrees of freedom where k is the number of classes. In case any class frequency is less than 5, adjacent classes will be pooled resulting in reduction in degrees of freedom.

Remark : The summation extends over all the k classes.

3. Understanding the song (for non Hindi readers) Aye Mere Watan ke logo through a free translation [3]

We would first understand the song *Aye mere watanke* logo with the help of its translation and with reference to the sequence of notes as provided in Table 3 before we move to the analysis part.

Line 1:*Ae mere watan ke logo, zara aankh me bhar lo paani,* [Translation **:** O' people of my country! Fill your eyes with tears! This line corresponds to notes 1- 20 in table 3].

Line 2 :*Jo shaheed huye hain unki, zara yaad karo qurbaani* [Translation :Remember the sacrifice, of those who became martyrs! This line corresponds to notes 22-41 in table 3].

Line 3 :*Tum bhoolnaa jao unko, isliye suno ye* kahaani [Translation :And lest you forget them, listen to this story.! This line corresponds to notes 42-60 in table 3].

Line 4*:Jo shaheed huye hain unki, zara yaad karo qurbaani* [Translation : Remember the sacrifice, of those who became martyrs!This line corresponds to notes 61-81 in table 3].

Line 5: *Jab ghayal huwa himalaya, khatre me padi azaadi, Jabtak thi saans lade woh, phir apni lash bichha di,*
[Translation: When the Himalayas were injured and our freedom was threatened,
They fought until their last breath , and then they laid down their bodies.Line 5 corresponds to notes 82- 127 in that sequence of table 3].

Line6: *Sangeen pe dharkar maatha, so gaye amar balidaani Jo shaheed huye hain unki, zara yaad karo qurbaani* [Translation: With their faces on their bayonets, the immortal martyrs went to sleep. Remember the sacrifice, of those who became martyrs! Line 6 corresponds to notes 128-166 in that sequence of table 3.]

Line 7: *Jay hind, jay hind ki sena* [Translation: JaiHind! Glory to the Indian Army! Line 6 corresponds to notes 167-188 in that sequence of table 3].

Line 8: *Jay hind, jay hind, jay hind* **[**Translation :Victory to India, Victory to India, Victory to India! Line 7 corresponds to notes 189-197 in that sequence of table 3].

Table 1 compares the Indian and western notes [1]. Table 2 represents the western notes in three octaves [1]. The note sequence of the song [2] are next provided in table 3.

Table 1: Representation of musical notes in Indian and corresponding Western notation [1]

S	r	R	g	G	M	m	P	d	D	n	N	INDIAN NOTATION
C	Db	D	Eb	E	F	F#	G	Ab	A	Bb	B	WESTERN NOTAION

S=Sa; r= komal (flat) Re; R= Sudh (natural) Re; g=komal Ga; G=sudh Ga; M=Sudh Ma; m=Tibra (sharp) Ma; P=Pa; d=komal Dha; D=sudh Dha; n=komal Ni; N-sudh Ni

Remark: Sa and Pa are always Sudh (natural). In the western notation, we have used the letter b to represent flat and the symbol # to represent sharp.

Table 2: Western notes in the three octaves

*C	*Db	*D	*Eb	*E	*F	*F#	*G	*Ab	*A	*Bb	*B	NOTES (LOWER OCTAVE)
C	Db	D	Eb	E	F	F#	G	Ab	A	Bb	B	NOTES (MIDDLE OCTAVE)
C*	Db*	D*	Eb*	E*	F*	F#*	G*	Ab*	A*	Bb*	B*	NOTES (UPPER OCTAVE)

Table 3: Note Sequence of "Aye mere watan ke logo"

Instance of realization of a musical note	Musical Note
1	D
2	G
3	G
4	G
5	G
6	A
7	G
8	F
9	F
10	F
11	D

12	G
13	G
14	G
15	G
16	G
17	A
18	G
19	F
20	F
21	D
22	F
23	A
24	A
25	A
26	Bb
27	A
28	G
29	G
30	A
31	B
32	B
33	A
34	A
35	G
36	G
37	F
38	A
39	G
40	G
41	G
42	D
43	G
44	G
45	G
46	G
47	A
48	G
49	F
50	F
51	F
52	D
53	G
54	G
55	G
56	G
57	A
58	G
59	F
60	F
61	D
62	F
63	A
64	A
65	A
66	Bb
67	A

68	G
69	G
70	A
71	B
72	B
73	A
74	A
75	G
76	G
77	F
78	A
79	G
80	G
81	G
82	D*
83	D*
84	C*
85	D*
86	D*
87	C*
88	B
89	A
90	G
91	A
92	D*
93	D*
94	D*
95	C*
96	D*
97	E*
98	E*
99	D*
100	C*
101	C*
102	D*
103	C*
104	B
105	G
106	A
107	A
108	C*
109	C*
110	B
111	A
112	G
113	B
114	A
115	A
116	G
117	A
118	A
119	Bb
120	Bb
121	A
122	Bb
123	A

124	F
125	A
126	G
127	G
128	D
129	G
130	G
131	G
132	G
133	A
134	G
135	F
136	F
137	D
138	G
139	G
140	G
141	G
142	A
143	G
144	F
145	F
146	D
147	F
148	A
149	A
150	A
151	Bb
152	A
153	G
154	G
155	A
156	B
157	B
158	A
159	A
160	G
161	G
162	F
163	A
164	G
165	G
166	G
167	D
168	C
169	D
170	D
171	C
172	F
173	F
174	E
175	G
176	F
177	F
178	D
179	C

180	D
181	D
182	C
183	G
184	G
185	F
186	A
187	G
188	G
189	F
190	F
191	G
192	F
193	F
194	G
195	F
196	F
197	G

4. Probabilistic Analysis

Our interest lies in statistically verifying whether the notes in the song are maintaining their probabilities or are the probabilities of notes varying over the instances of their realization. If the probabilities are fixed, as our experimental results will confirm, then the model is multinomial otherwise quasi multinomial.

Table 4 shows the probability of the notes which are used in the song and also their respective observed frequencies. The test which is used to verify if the note probabilities are being maintained in the song is Chi-square goodness of fit test. For that, the total number of notes are divided into approximately two equal parts. Since here the number of notes is odd i.e. 197, we conveniently take first sequential 99 notes to constitute the first part and the next sequential 98 notes to constitute the second part. If the overall note probabilities are being maintained in both the parts then the probability model is multinomial else quasi multinomial. Tables 5 and 6 give the note probabilities in the first half and the same in the second half of the song respectively.

Table 4: Probability of the notes for the whole song

Sl. No.	Note	Observed Frequency (O)	Probability = O/197
1	C	12	12/197
2	D	25	25/197
3	E	3	3/197
4	F	32	32/197
5	G	66	66/197
6	A	43	43/197
7	Bb	6	6/197
8	B	10	10/197
Total		197	1

Table 5: Probability of the notes for the first half (first sequential 99 notes of the song)

Sl. No.	Note	Observed Frequency (O)	Probability= O/99
1	C	3	3/99
2	D	15	15/99
3	E	2	2/99
4	F	14	14/99
5	G	36	36/99
6	A	22	22/99
7	Bb	2	2/99
8	B	5	5/99
Total		99	1

Table 6: Probability of the notes for the second half (last sequential 98 notes of the song)

Sl. No.	Note	Observed Frequency (O)	Probability= O/98
1	C	9	9/98
2	D	10	10/98
3	E	1	1/98
4	F	18	18/98
5	G	30	30/98
6	A	21	21/98
7	Bb	4	4/98
8	B	5	5/98
Total		98	1

Expected frequency is calculated using probability theory. Expected frequency is calculated for both first 99 and last 98 notes represented in table 7 and 8 respectively. The expected frequency of a note in a specific part (here first or second) is the overall note probability (table 4) multiplied by the total number of notes in the concerned part. Calculations of expected frequencies are shown in table 7 while table 7.1, based on table 7, shows the Chi-square calculations for the first half of the song.

Table 7: Expected frequencies for first 99 notes sequentially

Sl. No.	Note	Expected Frequency (E) = Probability of corresponding note from table 4 × 99
1	C	1188/197
2	D	2475/197
3	E	297/197
4	F	3168/197
5	G	6534/197
6	A	4257/197
7	Bb	594/197
8	B	990/197

Table 7.1: Chi square test for first 99 notes

O = Observed frequency; E = Expected frequency

Sl.No.	Note	O	E	O-E	$(O-E)^2$	$(O-E)^2/E$
1	C	3 15 } 18	1188/197 2475/197 } 3663/197	-117/197	0.3527274601	0.01897
2	D					
3	E	2 14 } 16	297/197 3168/197 } 3465/197	-313/197	2.524388673	0.14352
4	F					
5	G	36	6534/197	558/197	8.022984259	0.24189
6	A	22	4257/197	77/197	0.1527738411	0.00706
7	Bb	2 5 } 7	594/197 990/197 } 1584/197	-205/197	1.082867276	0.13467
8	B					

Calculated chi -square = $\sum(O - E)^2/E) = 0.54611$

Table Chi-square for 4d.f at 5 % level of significance = 9.49

Remark: We had 8 notes and accordingly our Chi-Square should be having 7 degrees of freedom (one degree of freedom is lost due to the restriction that the sum of observed and expected frequencies should be equal). However since some class frequencies were less than 5 classes had to be pooled resulting in 5 classes after pooling and hence 4 degrees of freedom.

Why pooling is necessary is because Chi Square is a continuous distribution and it cannot maintain its character of continuity if any class frequency (which is discrete) is less than 5.

Similarly, calculations of expected frequencies are shown in table 8 while table 8.1, based on table 8, shows the Chi-square calculations for the second half of the song.

Table 8: Expected frequency for last 98 notes

Sl. No.	Note	Expected Frequency (E) = Probability of corresponding note from table 4 × 98
1	C	1176/197
2	D	2450/197
3	E	294/197
4	F	3136/197
5	G	6468/197
6	A	4214/197
7	Bb	588/197
8	B	980/197

Table 8.1 Chi-Square test for last 98 notes

Sl. No.	Note	O	E	O-E	$(O-E)^2$	$(O-E)^2/E$
1	C	9	1176/197	597/197	9.1836687	1.53842
2	D	10	2450/197	-450/197	5.2178618	0.41955
3	E	1 18 } 19	294/197 3136/197 } 3430/197	313/197	2.52438867	0.14498
4	F					
5	G	30	6469/197	-559/197	8.05176634	0.24519
6	A	21	4214/197	-77/197	0.15277384	0.00714
7	Bb	4 5 } 9	588/197 980194 } 1568/197	205/197	1.08286737	0.13604
8	B					

Calculated Chi -square = $\sum(O - E)^2/E) = 2.49132$

Table Chi-square for 5d.f at 5 % level of significance= 11.07

Table 9 gives the transition probability matrix. The difference between the conditional and unconditional probabilities of notes confirms that the notes are not coming randomly. In fact, music is always planned and not random.

Table 9: Transition Probability Matrix of the notes of the song Aye Mere Watan Ke Logo

	C	D	E	F	G	A	Bb	B
C	2/12	5/12	0/12	1/12	1/12	0/12	0/12	3/12
D	9/25	6/25	1/25	3/25	6/25	0/25	0/25	0/25
E	0/3	1/3	1/3	0/3	1/3	0/3	0/3	0/3
F	0/32	7/32	1/32	13/32	3/32	8/32	0/32	0/32
G	0/65	4/65	0/65	14/65	34/65	12/65	0/65	1/65
A	1/43	1/43	0/43	1/43	20/43	12/43	5/43	3/43
Bb	0/6	0/6	0/6	0/6	0/6	5/6	1/6	0/6
B	0/10	0/10	0/10	0/10	1/10	6/10	0/10	3/10

Remark: To compute say P(A/B) we find the number of occurrences of B in the entire note sequences and then find out in how many cases B is followed by A. However if the last note is B then we have to subtract 1 from the denominator. This is because no information is available of the next transition for that note. In our case the last note is G. As there are 66 occurrences of G, so there will be 65 occurrences of G in which there is a succeeding note.

5. Interpretation

Both the Chi square values obtained are less than the corresponding table Chi square values at 5% level. Therefore, the overall relative frequencies (empirical probability) of the notes for the song of 197 notes are being maintained throughout the two parts of 99 notes and 98 notes. Hence the distribution followed by the notes of the song is multinomial.

6. Conclusion

Multinomial model is acceptable for the song *Aye mere Watan ke logo*.

Remark: Notes are statistically stable if they maintain their probabilities of occurrence over the instances of their realisation. There is another concept of pitch stability, based on note duration, which is psychological in character [1].

References

1. S. Chakraborty, G. Mazzola, S. Tewari and M. Patra, Computational Musicology in Hindustani Music, Springer International Publishing, Switzerland, 2014.
2. http://www.synthesizernotes.com/aye-mere-watan-ke-logo.html accessed on 29.06.2018
3. https://en.wikipedia.org/wiki/Aye_Mere_Watan_Ke_Logo accessed on 29.06.2018
4. https://en.wikipedia.org/wiki/Multinomial_distributionaccessed on 29.06.2018